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THE DEPARTMENT OF
THE ENVIRONMENT FOR NORTHERN IRELAND

Technical Memorandum (Bridges)
Shear Key Decks

Summary:

VOLUME 1 HIGHWAY STRUCTURES: APPROVAL PROCEDURES AND GENERAL DESIGN

BE 23

TECHNICAL MEMORANDUM (BRIDGES)

SHEAR KEY DECKS

Contents

SHEAR KEY DECKS

1. Over the past few years there has been an increasing interest shown in the use of pre-cast beams connected by shear keys for the design of bridge decks. The Department's original "Notes for Guidance in Checking Bridge slabs consisting of Shear Connected Beams" - Internal Memorandum No.34, Nov. 1965 - has therefore been rewritten to include the latest information on design methods and requirements, and is being issued as an Annexe to this Technical Memorandum.

2. The Annexe consists of three parts, namely: 1. Design Requirements, 2. Design Graphs, and 3. Worked Example. The complete Annexe is available free of charge upon request to Highways Engineering Computer Branch, Department of the Environment, St Christopher House, Southwark Street, London SE1.

3. The Annexe incorporates the design method developed by British Rail which allows the transverse bending stiffness of the joints to be taken into account and by doing so reduces the torsional strength requirements of the beams. However, when using this method of design special attention must be paid to the strength of the joints themselves.

4. In many cases when using the "stiff-joint" method of design, it will be found that the value of the reduction factor is such that when it is applied to the longitudinal torque in the beams and the transverse shear across the joints, these moments and forces are reduced to such low values that only nominal reinforcement is required to resist them. While it is not possible to lay down precise limits for the ranges of beam spans and widths when the longitudinal torsion and transverse shear calculations may be curtailed, the following table gives an indication as to when this may be assumed to hold good.

Width of Beam (mm)	Span (m)	Reduction factor
432	6.1	$\frac{1}{7.5}$
432	12.2	$\frac{1}{41.6}$
711	7.8	$\frac{1}{6.8}$
711	12.2	$\frac{1}{17.5}$
902	9.75	$\frac{1}{8.9}$

In most cases a reduction factor of about 1/7 will be found sufficient to reduce the longitudinal torsion steel and transverse shear steel requirements to nominal amounts. As the span is reduced and beam width increased the reduction factor is also increased and approaches closer to unity.

5. The table below gives an indication of the costs of decks constructed from hog-backed shear connected beams compared with other types of construction. For the comparison the costs per unit area of different types of deck have been shown as multiples of the cost per unit area of an RC in-situ slab. The costs for hog-backed beams were obtained from British Rail (Southern Region) and for the other types come from the Department's records for 1967, updated to 1969. It should be noted that the shear key decks were mainly designed for HA loading and checked for 30 units of HB.

Span Range (m)	Cost of Deck Comparison to RC Slab (in-situ)				
	RC Slab	Steel Beam and Slab	Concrete Beam and Slab	Solid Infilled PSC Inverted 'T' Beams	Hog Backed Shear Key
7.6/9	1	1.8	-	-	1.5
9/10.7	1	-	-	1.5	1.6
10.7/12	1	-	-	1.1	1.2
12/13.7	1	-	1.6	1.4	1.5
13.7/15	1	-	1.6	1.1	1.1*
15/18	1	1.3	1.8	-	1.8
18/21	1	1.1	1.8	-	-
21/24	1	0.9	-	-	1.4

*Bridge Constructed by Direct Labout

It will be seen from the table that the cost of shear key decks is comparable with other forms of pre-cast construction such as solid infilled inverted PSC 'T' beams. The average mid-span structural depth of hog-backed shear key decks is less than that of the sold infilled inverted PSC 'T' beam deck in the ratio 1: 1.3, for spans up to about 15m. Shear key decks have therefore much to commend them where construction depth is limited and where it is considered necessary to use a pre-cast method of Construction.

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27 November 1970

INTRODUCTION

1. This annexe contains the latest information on design methods and requirements for the design of bridge decks consisting of pre-cast beams connected by shear keys. It consists of three parts, namely:

1. Design Requirements,

2. Design Graphs,

and 3. Worked Example.

2. The Annexe incorporates the design method developed by British Rail which allows the transverse bending stiffness of the joints to be taken into account, and by doing so reduces the torsional strength requirements of the beams. However, when using this method of design special attention must be paid to the strength of the joints themselves.

3. In many cases when using the "stiff-joint" method of design, it will be found that the value of the reduction factor is such that when it is applied to the longitudinal torque in the beams and the transverse shear across the joints, these moments and forces are reduced to such low values that only nominal reinforcement is required to resist them. While it is not possible to lay down precise limits for the ranges of beam spans and widths when the longitudinal torsion and transverse shear calculations may be curtailed, the following table gives an indication as to when this may be assumed to hold good.

Width of beam (mm)	Span (m)	Reduction factor
432	6.1	1/7.5
432	12.2	1/41.6
711	7.8	1/6.8
711	12.2	1/17.5
902	9.75	1/8.9

In most cases a reduction factor of about 1/7 will be found sufficient to reduce the longitudinal torsion steel and transverse shear steel requirements to nominal amounts. As the span is reduced and beam width increased the reduction factor is also increased and approaches closer to unity.

4. Experience to date has shown that shear key decks can compare favourably in cost with other forms of pre-cast construction such as inverted PSC 'T' beams. There is also found to be a saving in construction depth, the average mid-span structural depth of hog-backed shear decks being less than that of the equivalent PSC 'T' beam deck in the ratio 1: 1.3, for spans up to about 15m. Shear key decks have therefore much to commend them where construction depth is limited and where it is considered necessary to use a pre-cast method of construction.

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1. INTRODUCTION

1.1 The purpose of this memorandum is to provide the designer with a desk method of analysing decks consisting of shear connected beams which has been developed by Dr. Spindel[1] of British Rail.

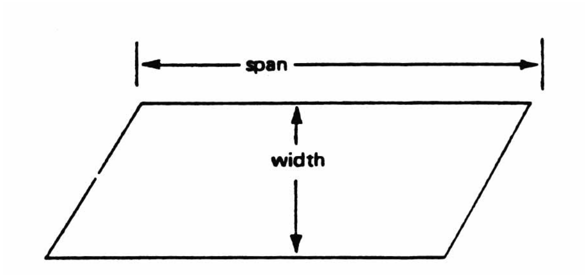
Originally the method assumed that the longitudinal joints, which are formed *in-situ*, had no stiffness and acted purely as hinges, distributing loads across the deck by shear action only. This assumption has been shown by test to be perfectly satisfactory[2], if somewhat conservative, and a considerable number of overbridges with spans up to 20m and span/depth ratios approaching 30 have been designed in this way and constructed by British Rail.

Compared with orthotropic deck slabs there is little difference in the maximum longitudinal bending moments, but the torques in the beams are much heavier and may be such that it is impracticable to reinforce them against this. This would be more likely in short, wide decks with wide units.

However, practical bridges have concreted joints which are not pure hinges. They are of appreciable depth with transverse reinforcement through them, which by hindering rotation reduces the torques, and tests on a model[2] suggest that torque is not likely to be a primary cause of failure.

A method of allowing for the reduction of rotation of the beams due to transverse bending stiffness in the joints has been developed[3,4]. This enables the reduced values of the torsion on the beams and the transverse shear in the joints to be calculated provided there is sufficient reinforcement to resist the transverse moments that are developed.

1.2 The analysis is based on the use of Fourier series for the evaluation of moments, shears and torsions in this type of deck. It is strictly only applicable to right bridge decks, but in a skew slab the bending moments will tend to decrease with increasing angles of skew, while the torques will increase. The effects of these two changes in the stress in a beam tend to cancel so that it is quite safe to design a skew bridge of up to about 30º as though it were a right bridge of span and width as indicated in the following sketch.



So far it has not proved possible to analyse skew bridges by any other method but further investigation will be carried out. Comparison of the hand method with an orthotropic plate program is given in Appendix 2 for the right deck used in the example in Part 3.

Only the first harmonic of each series is used and a set of graphs prepared on this basis forms Part 2 of this memorandum.

Appendix 1 amplifies the sections dealing with longitudinal bending moments and transverse shears.

1.3 Although decks carrying HA loading have in the past been designed on a strip basis it is recommended that such loading or any other should not be applied as a uniform loading to the whole of the deck but should be distributed by this (or any other suitable approved) method.

1.4 The Ministry wishes to acknowledge the work of British Rail on shear key decks and in particular that of Dr. J. E. Spindel and Mr. L. R. Waddington.

2. DESIGN LOADING

As BS 153 Part 3A: 1954 (read in conjunction with Technical Memorandum No BE 17) and MOT Memorandum No 771.

3. PERMISSIBLE STRESSES

As Technical Memorandum BE 10 (1st revision) and CP 115: Part 2: 1969 except that tensile bending stresses in the deck units shall not exceed $1 N/mm^2$ at the time between transfer and erection, and thereafter, except for residual stresses at the ends until the prestressing losses reach their calculated value, shall not be permitted.

4. LONGITUDINAL BENDING MOMENTS

4.1 The total longitudinal bending moments due to HA and HB live loading, road surfacing, pipe bays and parapets should each be calculated. (Moments due to HA and HB loading are included in the graphs in Part 2).

4.2 The moment on a beam due to any of these loads is obtained by dividing the total moment for each load by the total number of beams in the width of the bridge, and multiplying by the appropriate distribution co-efficient (which varies for each loading case). The total moment on the beam for any combination of loads is the sum of these individual moments.

4.3 In order to determine the distribution co-efficient it is necessary to calculate the first harmonic parameter β for the slab (all higher harmonics being neglected).

where $$\beta = \pi \times \frac{\text{width of slab}}{\text{span of slab}} \sqrt{\frac{EI}{GJ}}$$

Width and span of slab refer only to that part of the deck formed by the shear key beams and exclude edge stiffening, service bays and other forms of construction.

EI = flexural rigidity per unit width of slab.
GJ = torsional rigidity per beam divided by the width of the beam.
EI/GJ can be obtained directly from the first graph or calculated using published data for the torsional rigidity.

The co-efficients can then be read from the other graphs given in Part 2.

It should be noted that because of symmetry, values are only given for 5 of the 9 reference stations.

It is recommended that the influence lines of the distribution co-efficients for various transverse positions across the deck should be plotted rather than tabulating numerical values, except when edge stiffening is to be used when it may be preferable to use a tabulated method as in the Worked Example in Part 3.

4.4 The maximum longitudinal moment that can be induced in a beam in an unstiffened slab will occur when the centre of gravity of the load system is as close as possible to a longitudinal edge. The beam carrying this maximum moment will lie between these two positions; which beam it is will depend on the type of loading involved and how close, relative to the width of the slab, the centre of gravity is to this edge. The critical moment will usually arise from the combination of all loading systems acting simultaneously.

5. LONGITUDINAL SHEAR

5.1 Each beam shall be designed to withstand the longitudinal shear, torque and bending (if any) arising due to prestress, self weight and imposed loads. The shear and torque (see section 9) have to be considered together and the combination taken which (with prestress and bending stresses acting) gives the maximum principal tensile stress. This will normally arise near the end of the beam, where bending stresses from loading are small.

5.2 The longitudinal shear considered shall be that arising from the beam's own weight and the loads directly carried by the beam. In the case of HA loading this refers to the distributed and knife-edge loads acting over the width of the beam or to a 112kN wheel (or two of them if the beam is wide enough) if more severe.

5.3 The longitudinal shear due to that part of the parapets and pavements which is supported by the shear key deck may normally be considered to be resisted by the outer beams. Alternatively, the shear may be distributed through the whole slab, the value of this shear for any beam being obtained by dividing by the number of beams to give the average shear per beam and multiplying by the appropriate moment co-efficient.

6. TRANSVERSE SHEAR

6.1 The local transverse shear shall be taken as 0.41kN/m/kN of wheel load on spans from 0 to 8m with a linear reduction from 8m to a value of 0.16kN/m/kN of wheel load on a span of 27m.

6.2 The distributed shears are calculated according to section 6.4 and multiplied by the reduction factor given in section 6.5. The resulting values are added algebraically to the local transverse shear values, being positive if the wheel considered is on the same side of the joint as the wheel causing the local transverse shear and negative if on the other side.

6.3 Transverse shears will be greatest when the heaviest loads are entirely to one side of the joint and occupy the narrower width of bridge, with a minimum loading on the other side. Therefore, when checking for HB loading, no live loads should be taken on the lanes not carrying the abnormal vehicle since these will reduce the transverse shears. Similarly when HA loading is used in the design of the joints the case where only one outer lane is loaded will give the worst effect.

6.4 Values of distributed shear are calculated as follows:

6.4.1 For wheel loads

either:

(a) From the expression

$$\frac{2}{L} \sum P \frac{\sin \pi a}{L} [Q/P]$$

where

P is any wheel load on the bridge

a is the distance of the load from an abutment

L is the span

[Q/P] is the total distributed shear in a joint due to a unit load acting on the slab in the same way as the load P considered. It will depend on the transverse position of the joint and on the particular load and is obtained from the influence line graphs in Part 2 of this memorandum (for simplicity, the upper bound 2/L ΣP [Q/P] is sometimes used), or

(b) by equating the load per unit length to π^2/L^2 x M_{max}.,

where M_{max} is the maximum gross moment due to all the wheels loads, the maximum distributed shear being:

$$\frac{\pi^2}{L^2} M_{max} \text{ x average value of } [Q/P]$$

6.4.2 For distributed loads the equivalent expressions are:

(a) 4w/π x average [Q/P]

where w is the uniform load per unit length

(b) as 6.4.1 (b), M_{max} being in this case the maximum gross moment due to the distributed load.

6.4.3 Method (b) is shown in the worked example in Part 2, as it is slightly easier to use. The maximum transverse shear is produced with the load (s) in the position, longitudinally, which produces the max. BM.

6.5 The distributed transverse shear value calculated in section 6.4 assumes that the longitudinal beams are connected together transversely by pure hinges. However, if the concreted joints contain sufficient transverse shear reinforcement to restrict beam rotation the value of the distributed transverse shear can be reduced by the factor: 1/(1+K).

where

$$K = \frac{2L^2}{n^2\pi^2 B} \times \frac{A}{G}$$

L = span of bridge (metres)

B = effective width of beam (metres)

n = number of terms in Fourier series for the load (taken as n = 1)

A = transverse flexural rigidity of the beam, per metre width of beam (kNm)

G = torsional rigidity of the beam, per metre width of beam (kNm)

6.6 The shear reinforcement projecting into the joints shall be designed on the assumption that the total shear across a joint is resisted by direct tension in the stirrups of each adjacent beam.

Then if A_T is the amount of shear steel/per. beam/per. joint/per. metre required to resist transverse shear:

A_T = [local shear (kN/m) + distributed shear (kN/m)]/138 x 10^3 mm^2
(Steel stress applies to bars less than 40mm diameter)

Cold worked bars and hot rolled high yield bars may be used, subject to the requirements of Clause 1502 of Specification for Road and Bridge Works (1969), in shallow decks where the length of plain MS bars would not be sufficient to develop the required force.

The minimum reinforcement required in every beam to resist transverse shear in the joints shall not be less than 12mm dia. bars at 150mm centres.

7. TRANSVERSE MOMENTS

7.1 The reduction in distributed transverse shear at the hinges (6.5) and hence torque in the beams is dependent on the joint, treated as a reinforced concrete section, being sufficiently stiff.

Neglecting any top steel this stiffness is given by:

$$A = \frac{E_j d^2}{10^3 \times 3b_j} \left[n_j^3 d + \frac{3E_s}{E_j \times 10^3} \times A_{st} (1 - n_j)^2 \right] \text{ kN}$$

and since

$$G = \frac{E_b D^3}{12\phi^2 \times 10^6} \text{ kNm}$$

$$\frac{A}{G} = \frac{4\phi^2 d^2 E_j \times 10^3}{b_j D^3 E_b} \left[n_j^3 d + \frac{3E_s}{E_j \times 10^3} \times A_{st} (1-nj)^2 \right] \quad \text{per metre}$$

where

A_{st} = area of bottom transverse reinforcement through joint, mm²/mm
d = effective depth to bottom transverse reinforcement through joint, mm
$n_j d$ = depth to neutral axis of reinforced concrete joint, mm
b_j , taken as the actual width plus 50mm
D = depth of precast beam, mm
E_j = Young's Modulus of joint concrete, N/mm²
E_b = Young's Modulus of beam concrete, N/mm²
E_s = Young's Modulus of reinforcement, N/mm²
ϕ = EI/GJ of beam, obtainable from graph in Part 2

Values of E_j and E_b appropriate for live loading should be chosen and the effective width of the joint, b_j , taken as the actual width plus 50mm.

It is recommended that E_j should be given a lower value than E_b to allow for adverse placing conditions during construction.

7.2 The transverse bending moment that will be developed in the joint is given by:

$$m_n = \frac{2L}{n\pi} \times \frac{A}{G} \times T_o \times \frac{1}{1 + \frac{2L^2}{n^2\pi^2 B} \times \frac{A}{G}} \text{ kNm per metre run of joint}$$

$$= \frac{2L}{n\pi} \times \frac{A}{G} \times T$$

where T_o is the torque in kNm per metre width of beam, for hinged joint, and where T is the torque in kNm per metre width of beam for a stiff joint. The transverse reinforcement provided must develop the moment m_n within an acceptable stress. If the reinforcement provided for shear is not sufficient to do this it must be increased, and since the transverse steel is not continuous through the joint but, in effect, hooked into a longitudinal continuous beam formed out of the jointing material, both the strength of its anchorage and the strength of the longitudinal beam must be adequate.

8. BEAM ACTION IN TRANSVERSE JOINTS

The longitudinal bar at the bottom of the joint is subjected to forces from the transverse reinforcing bars and should be considered as a continuous beam loaded alternately on either side. The forces are spread by the concrete so that each load on the bar is distributed over an unknown length.

The following simple rule has been recommended that assumes a uniform distribution over a length l/2 for values of l not exceeding 300mm and allows for possible variations in the relative positions of the transverse steel in adjacent beams:

Maximum bending moment $= Wl/\alpha$
Maximum shear $= \beta W$
where W = maximum force in the transverse reinforcement
l = centres of the transverse reinforcement in each beam.
$\alpha = 16$, $\beta = ½$ if $l \leq 300$mm
or $\alpha = 10$, $\beta = 0.8$ if $l > 300$mm

9. TORQUE, AND REINFORCEMENT FOR LONGITUDINAL SHEAR AND TORQUE

9.1 All deck beams are subject to twist from non-uniform loading of the deck. The torque is greatest in the beam which has the greatest transverse shears of the same sign (+ OR -) along its edges.

9.2 The end torque in a beam, per unit width, is obtained from the transverse distributed shear by multiplying by span/π. It can be assumed that half this torque will act at midspan.

9.2.1 The total torque at the beam section is obtained by multiplying the average of the torques per unit width acting at its two sides by B (width of the beam), and the torsional shear stress is obtained by dividing the total torque by the torsional section modulus Z_T for the point being considered. These moduli are given for the mid-faces of the beam (where torsional shear stresses are at maxima), in terms of the bending modulus, on the first graph in Part 2.

9.3 Having obtained torsional shear stresses for appropriate loading combinations and having added the longitudinal shear stresses also acting (see section 5.1), the maximum principal tensile stress under the worst combination of loading (which should include prestress and self-weight and bending stresses, if any) should be determined for the critical beam.

If any value, after including the reduction factor referred to in section 6.5, exceeds the appropriate limiting value of principal tensile stress given in Table 1, the ends of the beam shall be provided with closed stirrups to resist the maximum effects of longitudinal shear and torque combined. Principal tensile stresses in excess of twice the values in Table 1 are not allowable. Along the beam, closed stirrups to resist the maximum effects of longitudinal shear and torque combined shall be extended to a distance at least equal to the depth of the beam beyond the point at which the maximum principal tensile stress reduces to the limiting value as described above.

The area of stirrup steel per unit length of beam required to resist the longitudinal shear shall be that given by the following formula:

$$A_{ss} = \frac{V}{P_{st}\, F_{HB}\, d_t} \times 10^3 \text{mm}^2/\text{m}$$

where V = longitudinal shearing force, N.
P_{st} = permissible tensile stress in stirrups
= 138N/mm^2 (for bar less than 40mm diameter).
F_{HB} = 1.25 where Type HB loading or 112 kN wheel loads have been included.
= 1.00 in all other cases.
d_t = depth of prestressing tendons at section considered, mm.

The area required may be given in any number of vertical legs not less than two per stirrup station.

The area of transverse steel per unit length of beam to resist torque may be calculated (for beams with a substantial prestress) for each face from the following formula:

$$A_{st} = \frac{T^2 C}{6\ \sigma\ P_{st}\ F_{HB}} \times 10^3 \text{mm}^2/\text{mm}$$

where T = torsional shear stress for the midpoint of the face considered N/mm^2.
o = average longitudinal prestress at the section N/mm^2.
C = beam dimension measured at right angles to face considered mm.
P_{st}, F_{HB} = as for A_{ss} above.

The size of the torque reinforcement should be determined by the maximum of the areas so obtained, and the stirrups should be closed links as in reinforced concrete columns. Substantial longitudinal bars should also be provided in any corners where the prestress is of low value.

9.4 For most prestressed bridge decks, especially those of appreciable span having narrow beams, the above provisions will probably result in a nil requirement of stirrups. Nevertheless, all bridge beams shall be provided with closed stirrups throughout their length at centres not exceeding three-quarters of the beam depth.

Calculations made in accordance with the American Code ACI 318/63, Section 2610, may be used to determine the minimum quantities needed to meet the ultimate load requirements of CP. 115, C1.331, last paragraph. In no circumstances should the quantity provided be less than 0.1% of the horizontal area of the concrete at the section under consideration.

9.5 For a comprehensive treatment of the design of reinforced and prestressed concrete beams subject to shear and torsion reference may be made to a book by H.J. Cowan[5].

Table 1

Specified Works Cube strength for concrete(N/mm^2)	Principal tensile stresses (N/mm^2)
30	0.85
40	1.0
50	1.2

10. EDGE STIFFENING

10.1 For a thin slab of flexural rigidity 2bF connected to edge stiffening beams of flexural rigidities EI_A and EI_B at their centroidal axes and then loaded with a longitudinal BM of value M applied anywhere, the bending moments induced in the edge beams are given by:

$$M_A = M \times \frac{K_A S_B - K_B K_{1,2}}{S_A S_B - (K_{1,2})^2} \qquad (1)$$

$$M_B = M \times \frac{K_B S_A - K_A K_{1,2}}{S_A S_B - (K_{1,2})^2} \qquad (2)$$

Where $K_{1.1}$ = distrib. coeff. for an edge of the slab with a unit load applied at the edge, if the slab were unstiffened.

$K_{1.2}$ = distrib. coeff. for an edge of the slab with a unit load applied at the opposite edge, if the slab were unstiffened.

$$S_A = \frac{2bF}{EI_A} + K_{1.1} \qquad (4)$$

$$S_B = \frac{2BF}{EI_B} + K_{1.1}$$

K_A = distrib. coeff. for edge A of the slab under the moment M, if the slab were unstiffened.
K_B = distrib. coeff. for edge B of the slab under the moment M, if the slab were unstiffened.

If there is no stiffening beam at edge B, then $M_A = MK_A/S_A$ (5)

If there is no stiffening beam at edge A, then $M_B = MK_B/S_B$ (6)

If the edge beams are connected to the slab at top or bottom instead of at their centroidal levels, then they will have increased effective stiffness as a portion of the slab will tend to act with them like the flange of an L beam. If desired, a width of slab not exceeding one-sixth of its span may be included in the calculation of EI_A and EI_B but the second moment of area of that part of the slab about its own neutral axis should be excluded. The flexural rigidity of the slab should remain unaltered at 2bF.

10.2 The above expressions are derived from deflection compatibility and are thus, strictly, only applicable if the connections of the beams to the slab are in the nature of hinges - however, any effects arising from slope incompatibility are likely to be small and can be neglected. They are valid for slabs of any type, and can be applied to deflections, loads and shears (with some loss of accuracy) as well as bending moments.

Depending on the relative values of $K_{1,1}$,$K_{1,2}$,K_A and K_B it is sometimes possible to use equation (5) instead of equation (1) and equation (6) instead of (2) for determining the edge beam moments.

10.3 The bending moments per unit width in the slab, at the various stations can be found by analysing the unstiffened slab under the loading (or loadings) M - in the course of which K_A and K_B are found - and then superimposing the effects of hogging bending moments M_A and M_B at the edges.

10.4 The maximum transverse shear and maximum torque that can be induced in a beam in a slab will occur when the centre of gravity of the load system is as close as possible to the (weakest) longitudinal edge. The maximum longitudinal moment in a beam may occur with the load in this position or when the centre of gravity of the load systems is on the longitudinal centre line, depending on the relative stiffness of the edge beams compared with that of the slab.

11. HOLLOW BEAMS

11.1 The methods outlined so far in this memorandum were developed for solid precast beams with no *in-situ* structural concrete other than in the joints. Increasing use is being made of voided and box beams with no diaphragms or other internal stiffening.

The resultant problems of what torsional stiffness to allow and how to know when the effect of warping or shear deformation becomes important, are not peculiar to shear key deck construction, although they may be more significant here than in more monolithic forms of construction. Research is being carried out on these aspects and when results are available information will be circulated, but for the present the assessment of designs when the void ratio is more than the nominal "ignorance" levels given in 11.2 and 11.3 must be left to the judgement of the engineer concerned.

Where the beams are subsequently covered with a well keyed layer of *in-situ* structural concrete the possibility of warping or shear deformation is clearly diminished. Such a slab should be assessed to act compositely with the beams in bending and separately in torsion.

11.2 Interim recommendations for box beams without internal diaphragms are as follows:

11.2.1 The void ratio depth of void/depth of beam or width of void/width of beam (whichever is the lesser) should not exceed 0.5.

11.2.2 Torsional inertia

$$J \quad \frac{4A^2}{\phi ds/_t} \quad \text{or} \quad \frac{4A^2 t}{S}$$

Where A = area enclosed by the median line (mm^2).
ds = distance measured along the median line (mm).
t = wall thickness (mm).
S = total perimeter (uniform thickness) (mm).

11.2.3 Torsional stress = Torque/2At (N/mm^2) (Torque in Nmm)

11.2.4 Torsional reinforcement when required (see 9.2),

$$A_{st} = \frac{\text{Torque}}{l \times 138} \quad (mm^2)$$

where l = depth or width (centre of reinforcement) whichever is the lesser (mm).

11.3 Similar recommendations for circularly voided beams without internal diaphragms are:

11.3.1 The void ratio (diameter of void)/(depth of beam) should not exceed 0.6.i

11.3.2 Torsional inertia as 11.2.2.

11.3.3. Torsional modulus of beam

$$Z_T = \frac{2J}{D}$$

11.3.4 Torsional reinforcement as required in 9.3.

12. WORKMANSHIP IN TRANSVERSE JOINTS

12.1 The sides of the units shall be roughened, with aggregate exposed.

12.2 The *in-situ* concrete in the joint shall have a 28-day strength which is not less than that of the precast units.

12.3 In order to ensure adequate compaction of the concrete in the joint the minimum throat width between units shall not be less than 75mm.

12.4 In bridge reconstruction and widening schemes, the need to maintain traffic may sometimes make it necessary to construct the new deck in portions. Where transverse mild steel reinforcement is made continuous through these joints, movements of this steel must be prevented for sufficient time, and to a sufficient extent, to allow the newly placed joint concrete to develop adequate bond and strength.

Where such a portion of deck is to be added to that already used by traffic the following requirements are called for:

(a) Rapid hardening Portland Cement should be used for the joint concrete.

(b) The minimum traffic restrictions which should be imposed on the use of the carriageway lanes adjacent to newly formed joints are as follows:

1. During the placing of joint concrete, and for a period of 48 hours thereafter, traffic to be restricted to vehicles having a gross weight not exceeding 2 tons in weight. Such traffic to be limited to crawl speed and kept as far removed as possible from the line of the joint. It is of course very much better, wherever practicable, for traffic to be excluded entirely during this period.

2. For the next following period of 7 days the carriageway may be used by vehicles which comply with Construction and Use Regulations provided they are restricted to crawl speed.

3. Vehicles of a gross weight greater than that permitted by C & U regulations to be prohibited from using the carriageway during the forming of a joint and for a period of three weeks thereafter.

Where rubber bearings are provided temporary packings should be inserted so as to eliminate vertical movement at the supports.

Alternatively, provided that the connecting joint is left out, the rest of the joints in the new portion may be concreted in the normal way, a minimum of two weeks being left before the final joint, which is then subject to the above requirements, is made.

13. REFERENCES

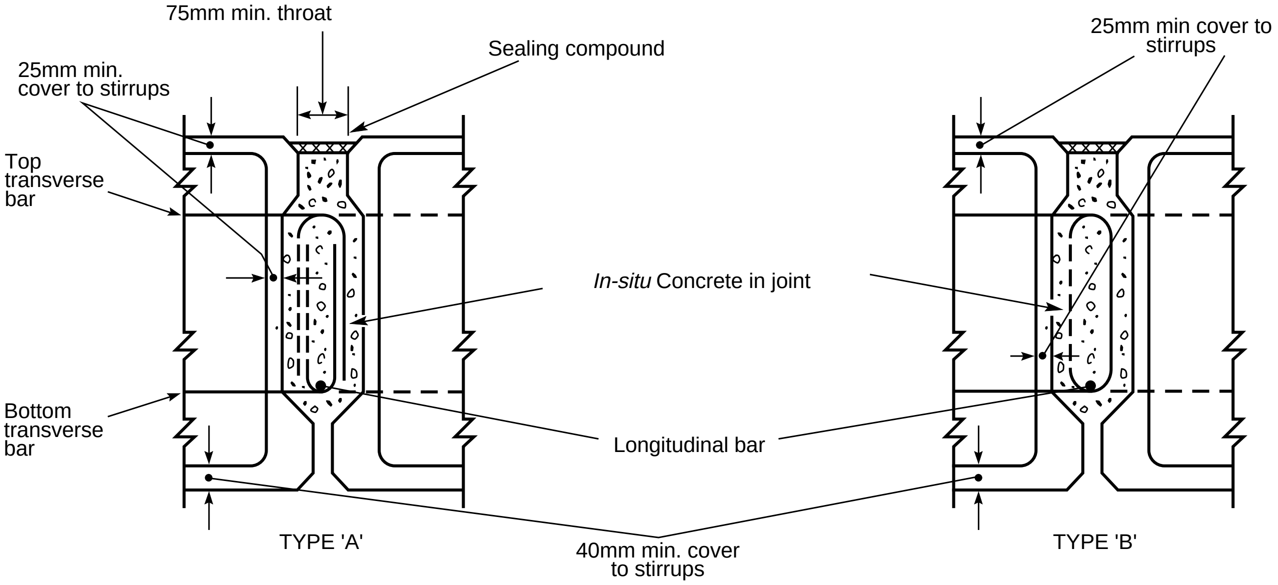
1. "A Study of Bridge Slabs having no Traverse Flexural Stiffness", by J.E. Spindel, PH.D. Thesis, 1961, Kings College, London.

2. "Tests of a Prestressed Concrete Bridge incorporating Transverse Mild Steel Shear Connectors", by B.C. Best, C & CA Research Report 16.

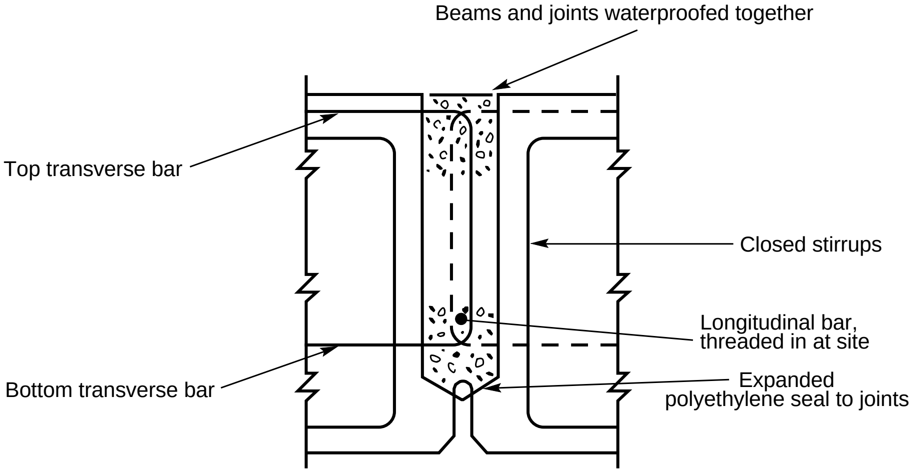
3. Contribution by Dr Spindel to discussion of Cusens and Pama's Paper "Edge Beam Stiffening of Multi-beam Bridges" Proc.AM.Soc.CE.Jan 1968.

4. "Torsion in Decks of Shear Connected Precast Beams" British Rail Internal Note (copies available from MOT).

5. "Reinforced and Prestressed Concrete in Torsion", by H.J. Cowan, Arnold (1965).



Note: Top transverse bars to be as high as possible to control shrinkage and cater for hogging. They should not be more than about 8mm dia. so that they can be bent by hand if necessary to facilitate the placing of the longitudinal bar.

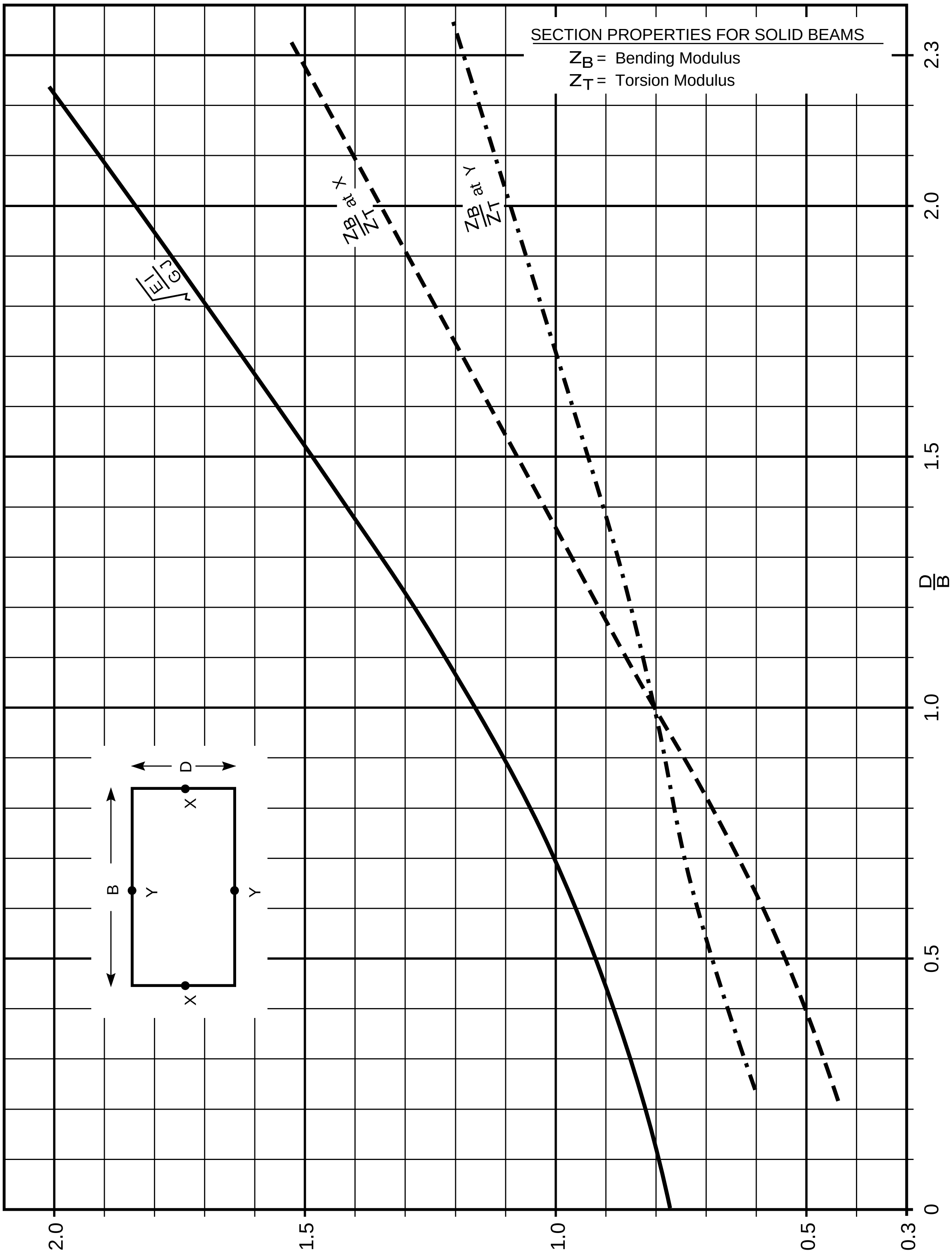


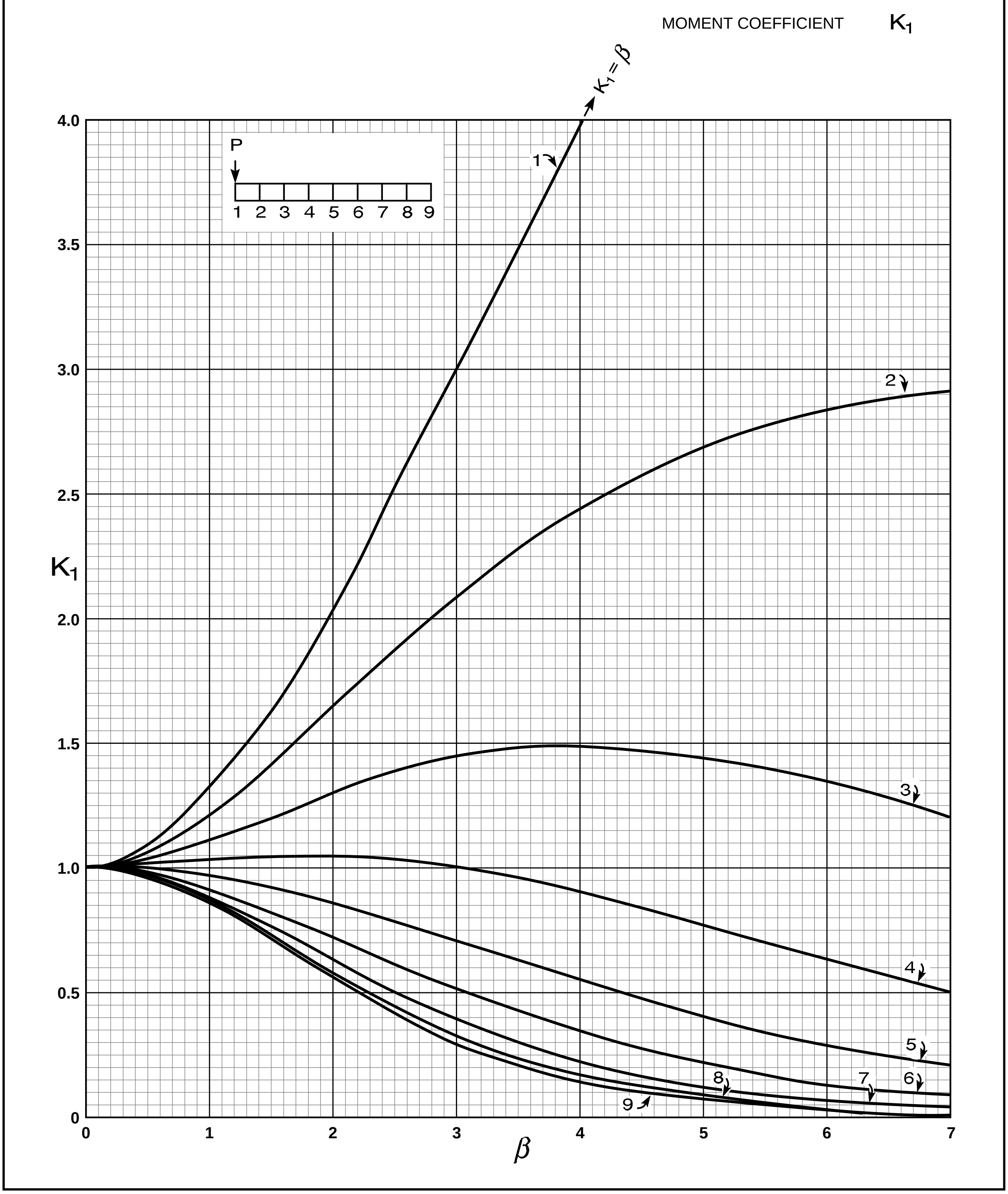
DETAILS OF JOINT USED BY BR (Southern Region)

TYPICAL JOINTS

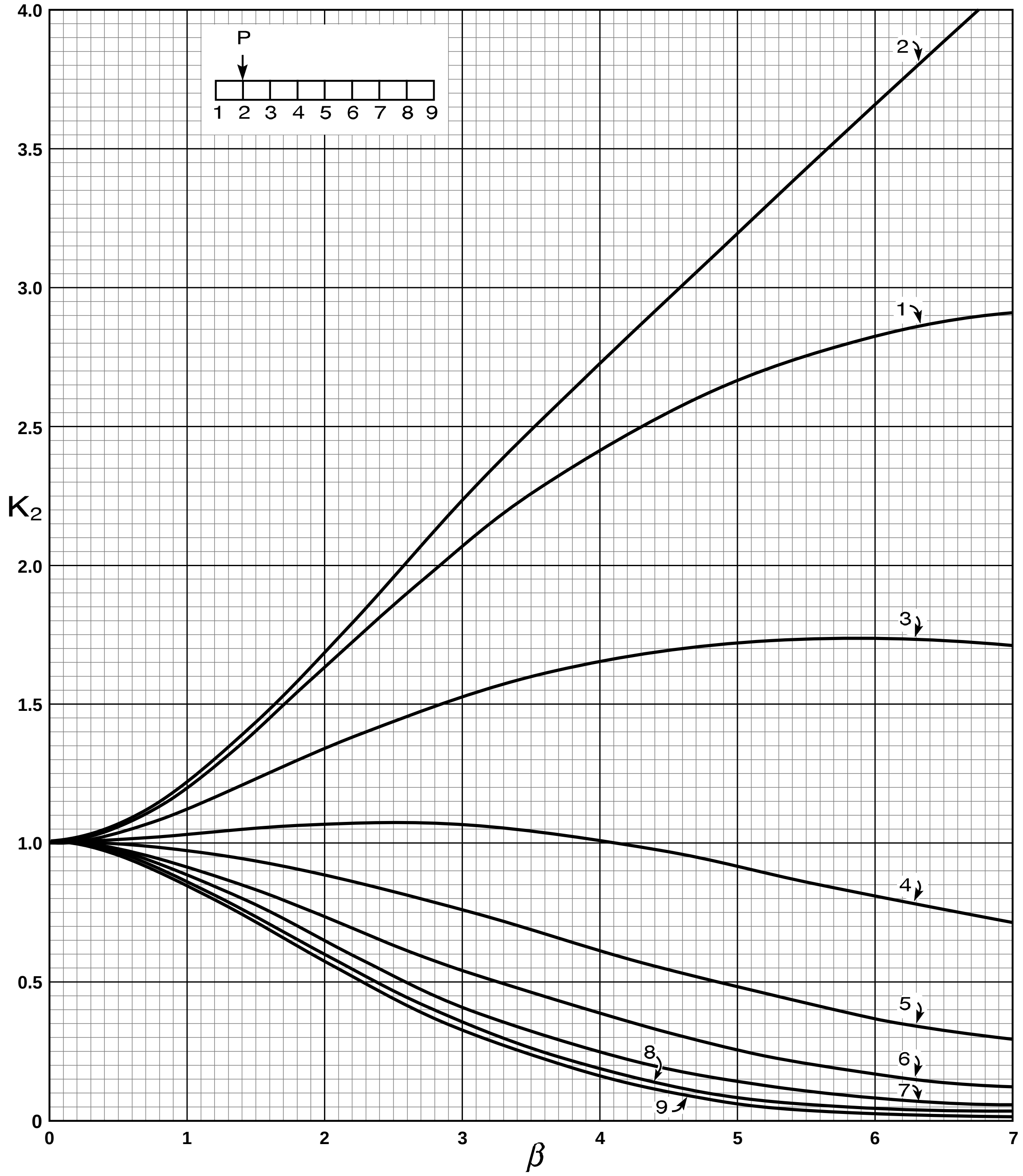
PART 2 DESIGN GRAPHS

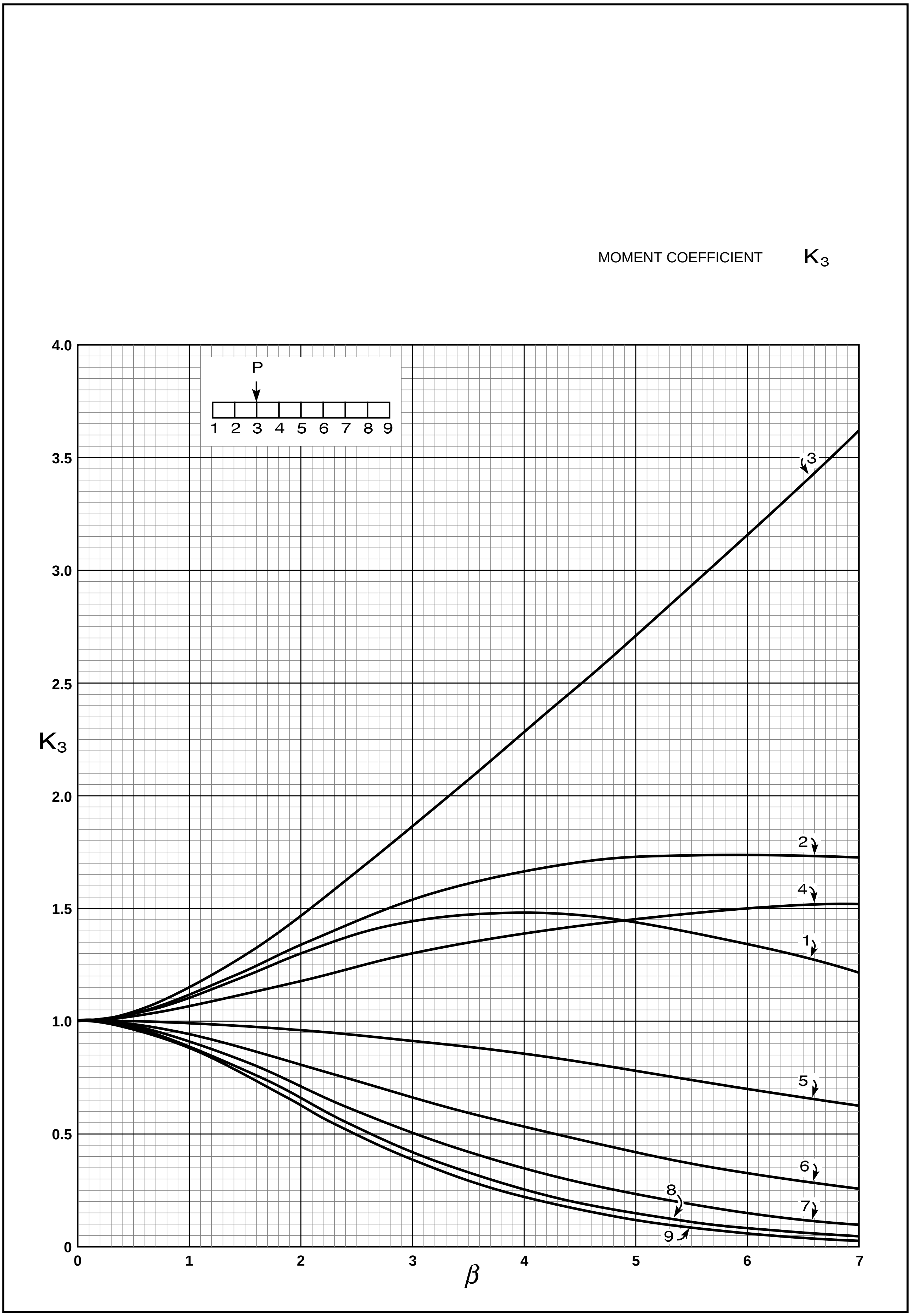
Values of the section constants required for the calculation of the parameter β and the influence lines for the distribution coefficients and transverse shears.




MOMENT COEFFICIENT K_1
$K_1 = \beta$
P
1 2 3 4 5 6 7 8 9
K_1
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
1
2
3
4
5
6
7
β
1
2
3
4
5
6
7
8
9

MOMENT COEFFICIENT K_2

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P
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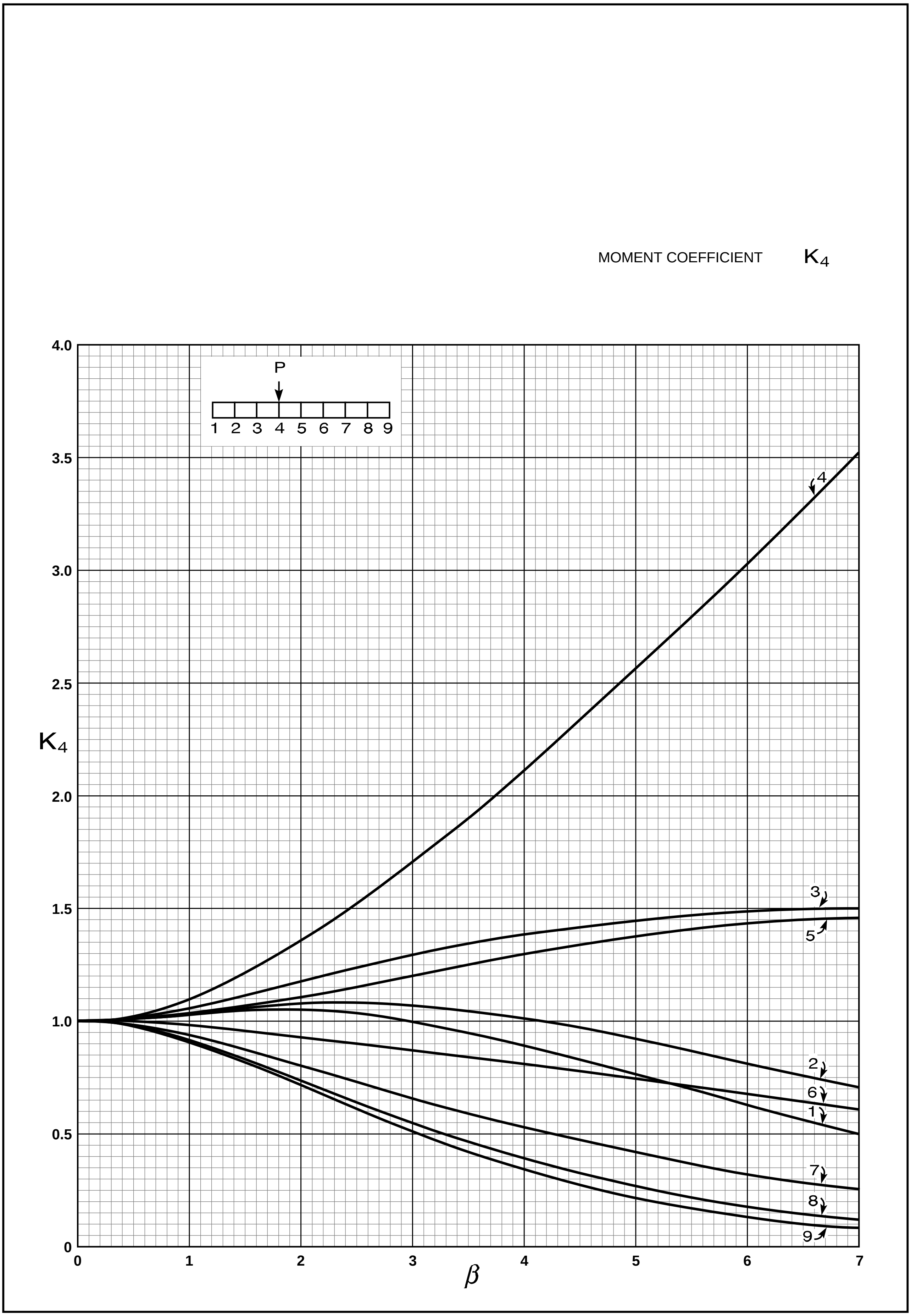
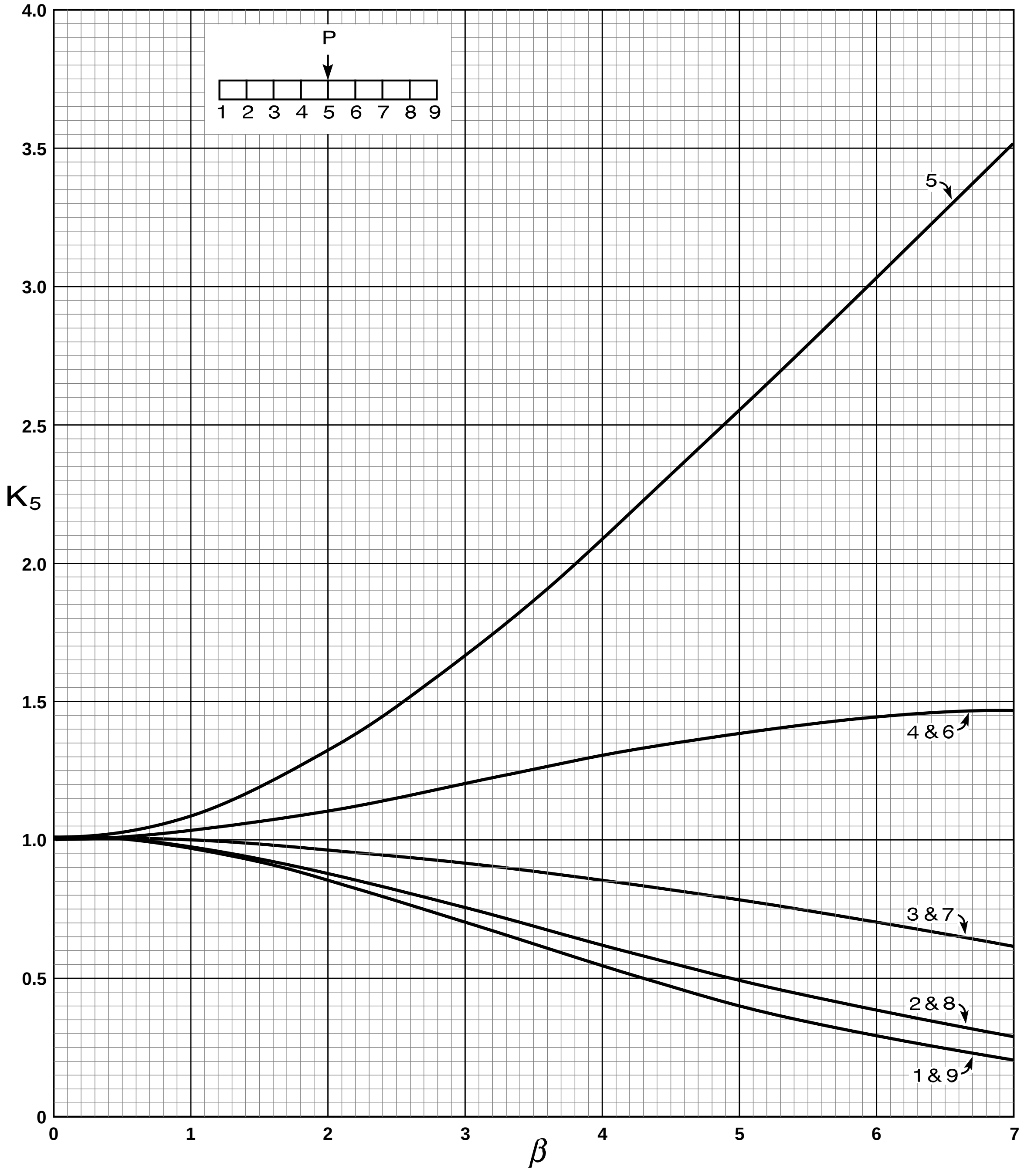
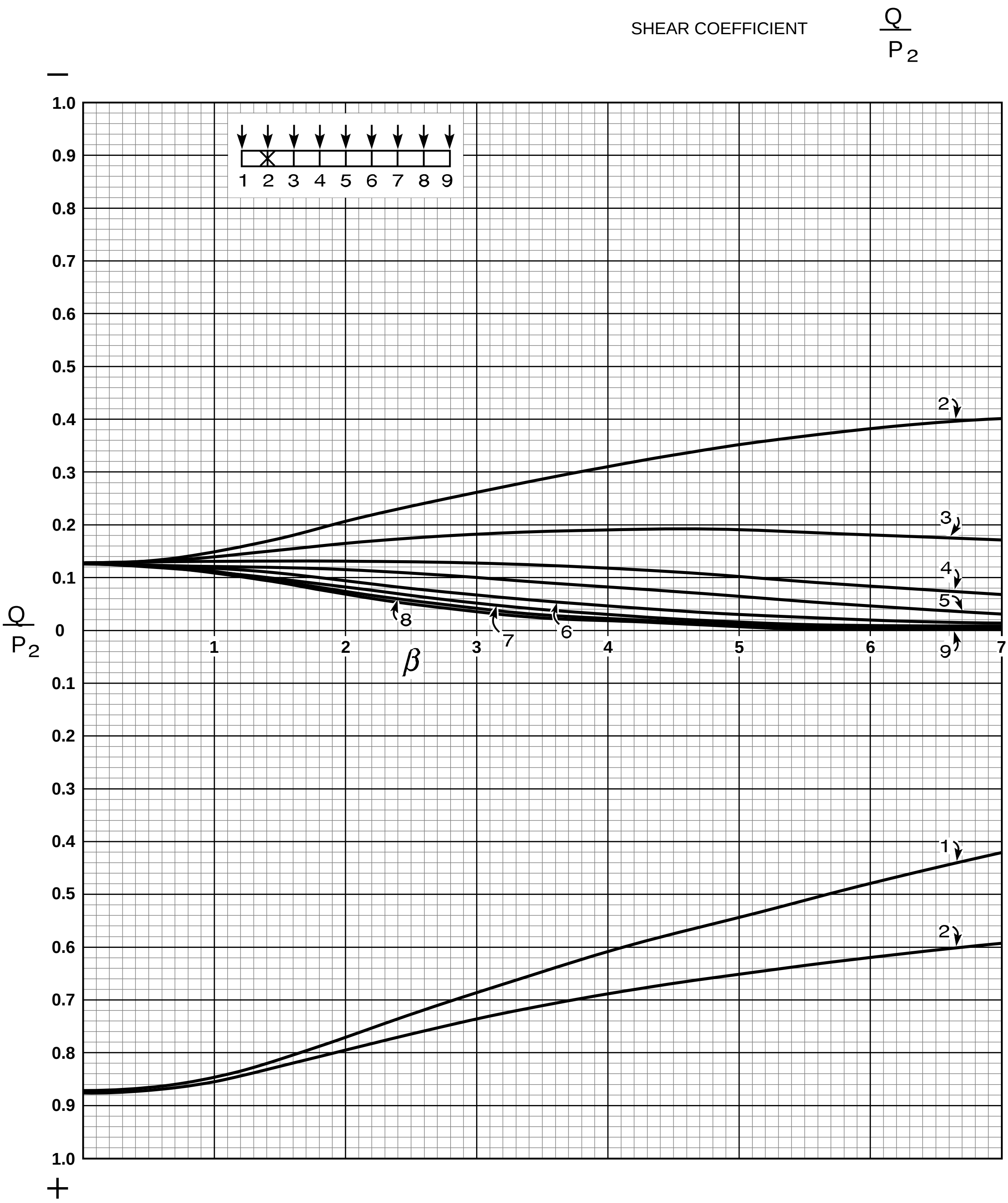
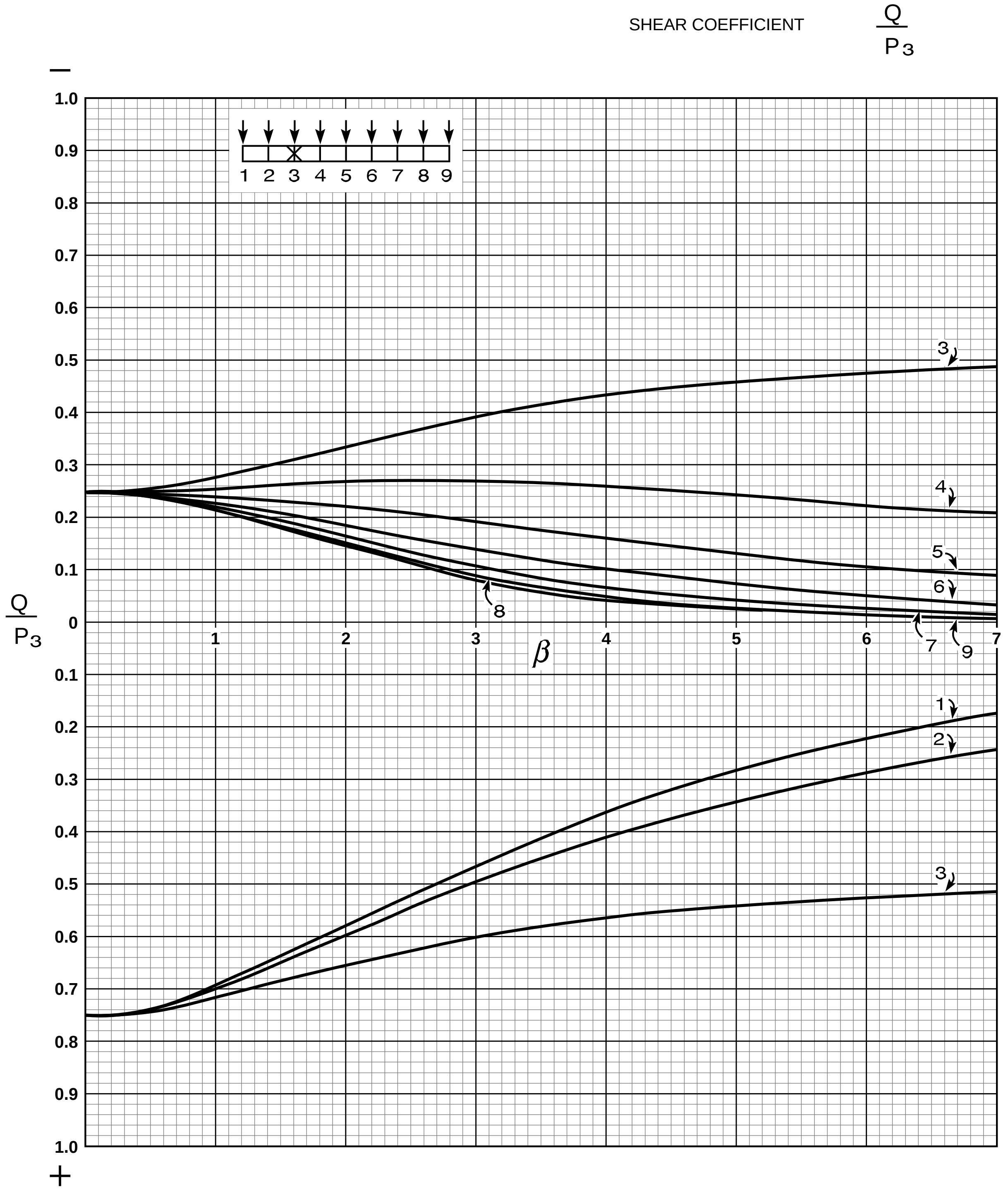
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
K3
0 1 2 3 4 5 6 7
β
3
2
4
1
5
6
7
8
9

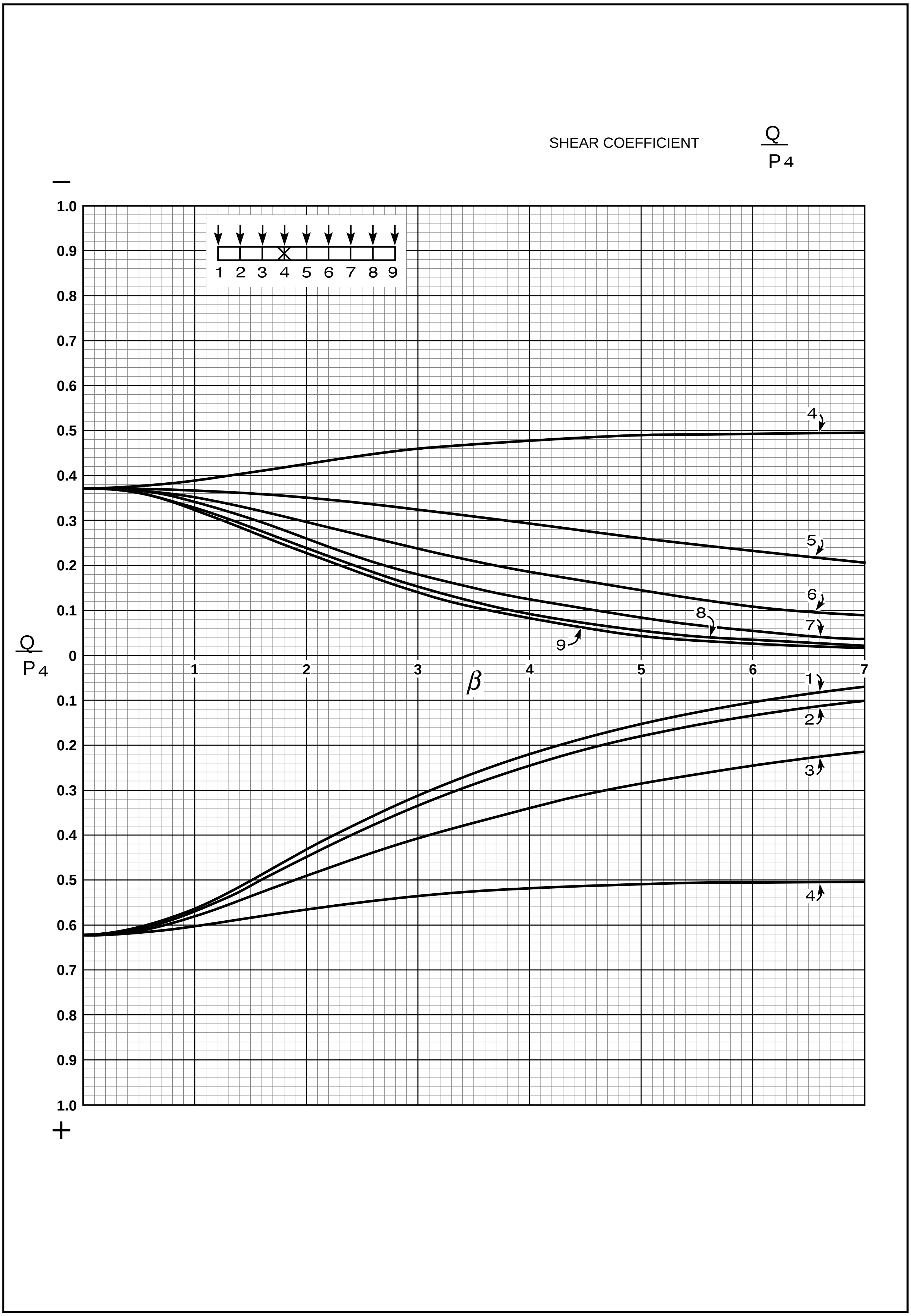

MOMENT COEFFICIENT K4
K4
P
1 2 3 4 5 6 7 8 9
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
0
1
2
3
4
5
6
7
β
4
3
5
2
6
1
7
8
9

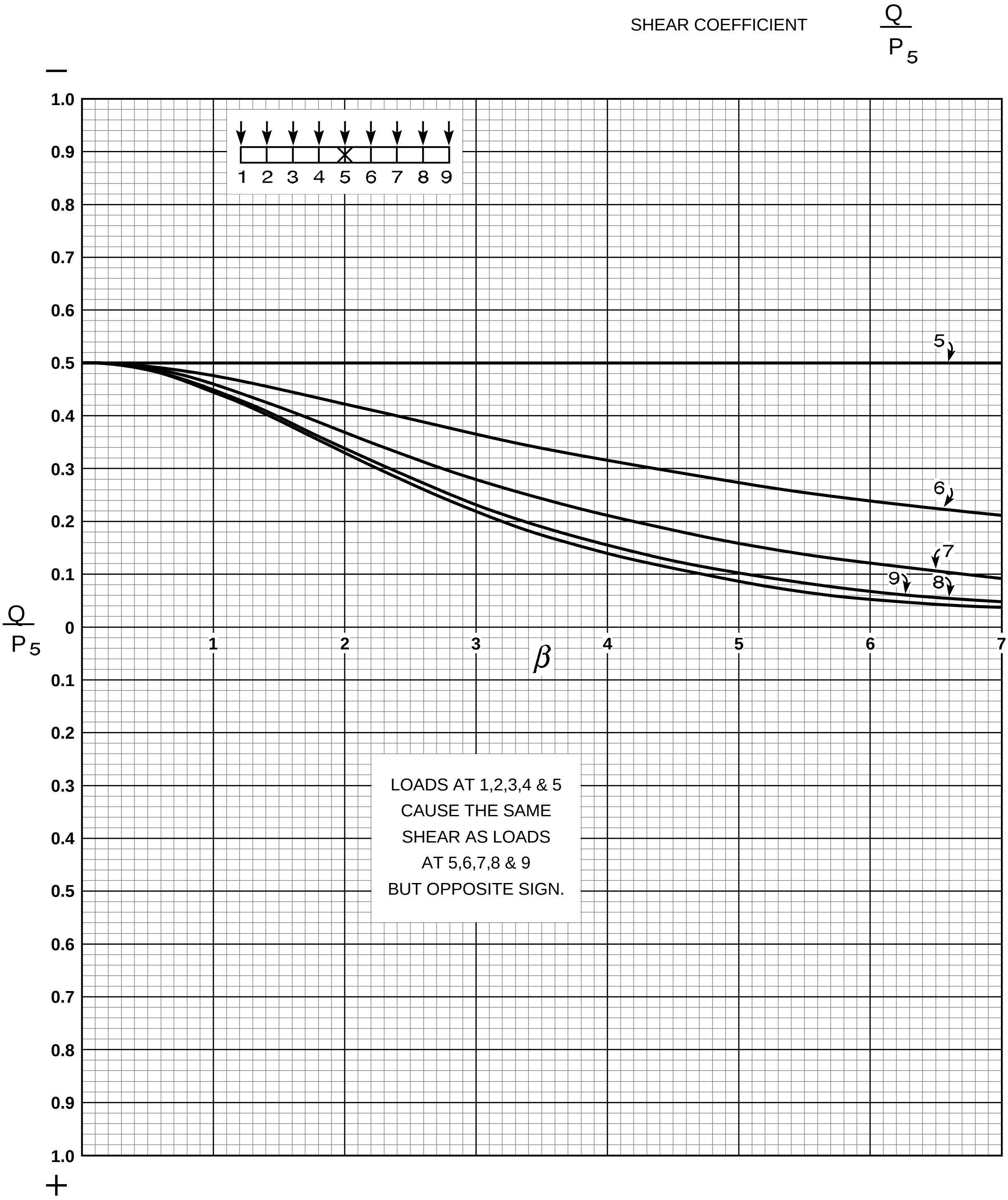
MOMENT COEFFICIENT K_5

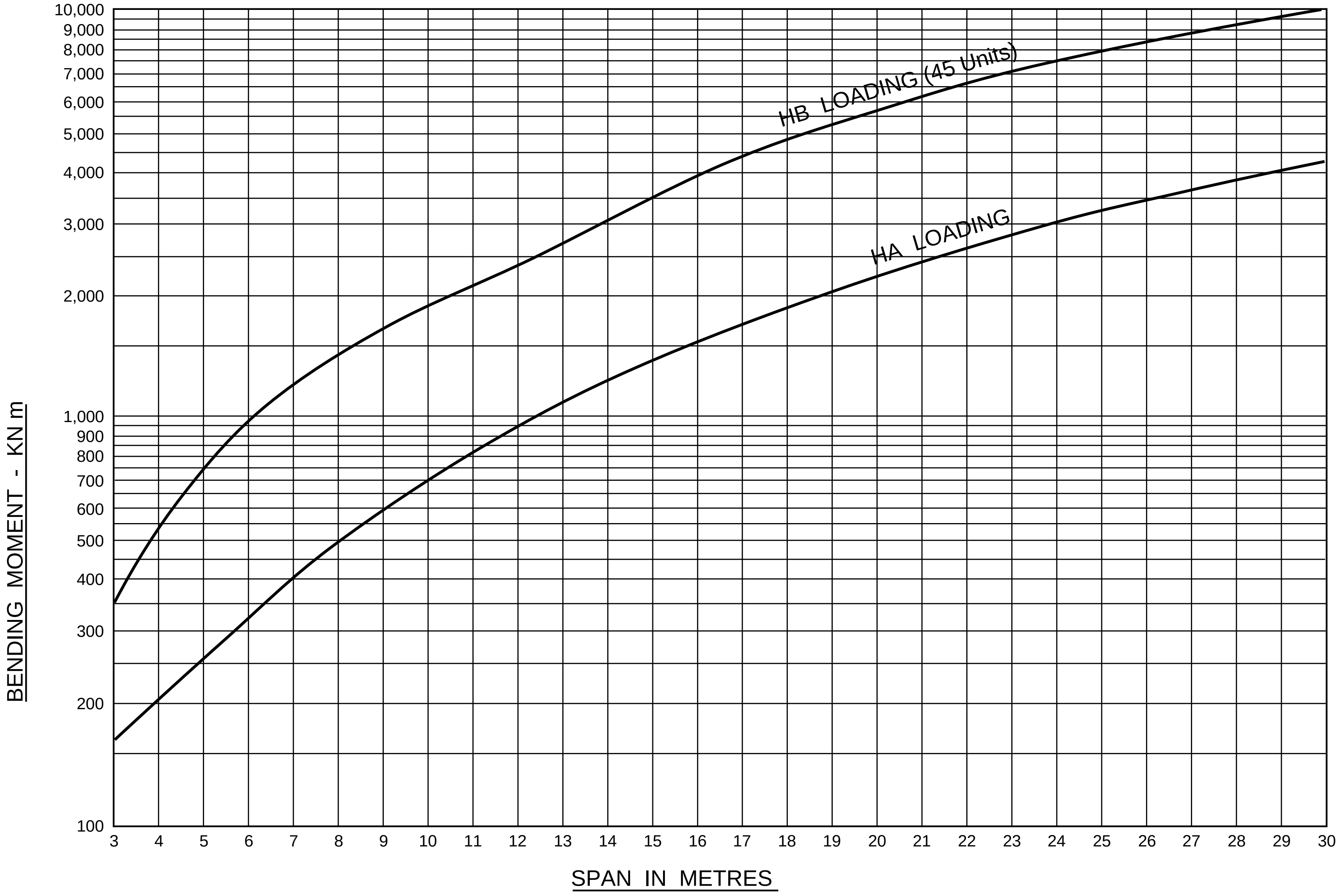



SHEAR COEFFICIENT Q/P2
1 2 3 4 5 6 7 8 9
Q/P2
β
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
7
8
9
−
+


SHEAR COEFFICIENT
Q / P3
–
+
1 2 3 4 5 6 7 8 9
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1 2 3 4 5 6 7
β
Q / P3
3
4
5
6
7
8
9
1
2
3


SHEAR COEFFICIENT
Q / P4
–
+
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
7
8
9
β


SHEAR COEFFICIENT $\frac{Q}{P_5}$
−
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$\frac{Q}{P_5}$
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
+
1 2 3 4 5 6 7 8 9
1
2
3
4
5
6
7
β
5
6
7
9
8
LOADS AT 1,2,3,4 & 5
CAUSE THE SAME
SHEAR AS LOADS
AT 5,6,7,8 & 9
BUT OPPOSITE SIGN.



LONGITUDINAL BEAMS HA LOADING & HB LOADING FOR ONE LANE

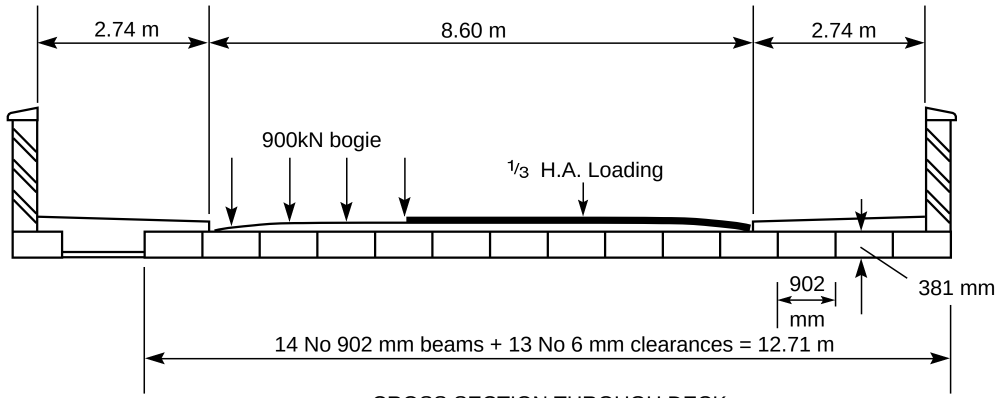
1. JOINT ACTING AS PURE HINGE

The following example is intended to illustrate the use of the distribution coefficients given in the charts or tables, and some effects of introducing edge stiffening. It does not include all the loadings which might occur in a bridge deck.

Note: (1) In the calculation of stresses due to loads applied after the concrete in the joints has reached minimum specified cube strength the cross section of a beam has been taken to be that of the smallest rectangle which will just enclose the precast unit.

(2) Any part of the loading carried by the slab before edge stiffening becomes effective must be treated as loading on the unstiffened slab - e.g. part of parapet forming edge stiffening, unless the edge is propped until hardened.

Span of bridge: 9.75m
Width of bridge: As cross section below.
Loads considered: 45 unit HB vehicle, 1/3 HA loading on remaining road width, road surfacing at $2.3kN/m^2$, "services, etc." load at L.H. edge of slab at 7.3kN/m and "parapet" load at R.H. edge of slab at 14.6kN/m.



CROSS SECTION THROUGH DECK

Longitudinal Bending

Gross max longitudinal bending moments on slab.				*Loading type*
due to 45 units HB (read from graph, Part 2)	=	1810kNm	= M	A
due to HA in one lane (read from graph, Part 2)	=	680kNm		
therefore due to 1/3 HA in 1.8 lanes	=	400kNm	= 0.22 M	B
due to road surfacing 2.3 x 8.60 x $9.75^2/8$	=	240kNm	= 0.13 M	C
due to "services" etc 7.3 x $9.75^2/8$	=	87kNm	= 0.05 M	D
due to "parapet" 14.6 x $9.75^2/8$	=	173kNm	= 0.10 M	D

Loading Cases considered

Case I

The HB vehicle will be taken near L.H. kerb, as shown in the cross-section of the deck, with HA/3 loading to its right, and;

Case II

The HB vehicle will be taken central on slab (nearly) with HA/3 loading to each side of it

Inspection of the influence curves in Part 2 indicates that no load anywhere on the deck ever produces negative values of K so Cases I and II will each include the road surfacing, services, etc., and parapet, etc. loads.

Cases IA and IIA will be with edge stiffening non-effective.

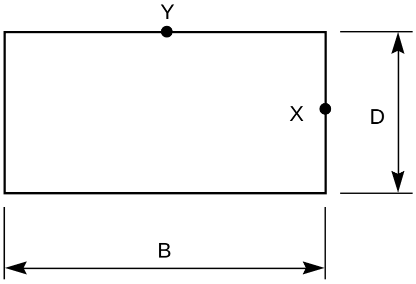
Cases IB and IIB will be with edge stiffening effective.

It will be assumed that the "parapet" dead load and the "services, etc." dead load are carried by the unstiffened slab and that the edge stiffening considered is derived from the bonding, on hardening, of appropriate parts of these items to the deck.

For the stiffened case, the road surfacing will be assumed applied after the stiffening has become effective.

Beam Properties (effective)

D = 381 mm
B = 902 mm
∴ D/B = 0.422



Hence, from Graph (Part 2)

$$\frac{EI}{GJ} = 0.885$$

∴ Z_B/Z_T at X = 0.505 and Z_B/Z_T At Y = 0.655

where $Z_B = 902 \times 381^2/6 = 2.18 \times 10^7 mm^3$

∴ Z_T at X = 4.33×10^7 mm^3 and Z_T at Y = 3.33×10^7 mm^3

Slab Properties (effective)

Width 2b = 12.71m
Span 2a = 9.75m

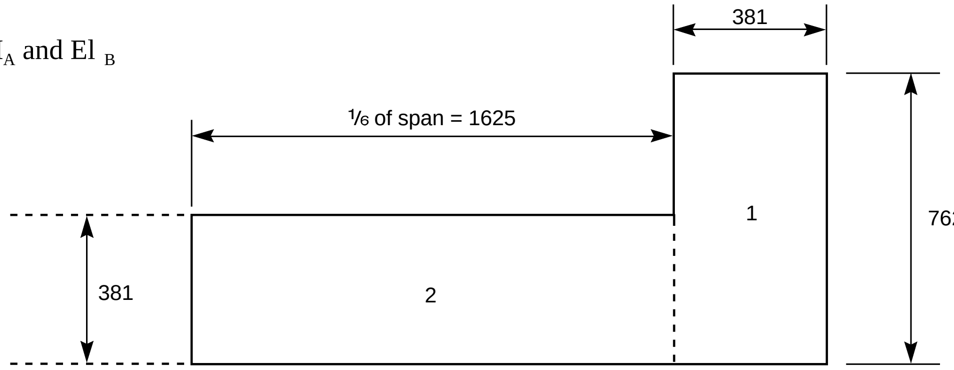
∴ $\beta = \pi \times 12.71/9.75 \times 0.885 = 3.62$ (see 4.3)

$K_{1.1}$ = 3.60 and $K_{1.2}$ = 0.18
2bF = $12710 \times 381^3/12E = 5.86 \times 10^{10} \times EN.mm^2$ (see 10.1)

Edge Stiffening Properties (see 10)

Beam 381mm x 762mm will be considered to be added at the edges of the slab. Edge A will be the "services" side and Edge B, the "parapet" side.

Calculation of EI_A and El_B



Item	*Size*	*A*	*y*	*Ay*	*y^2-y*	*$A(y_2-y)^2$*	*l self*
(1)	381 x 762	2.90×10^5	381	1.11×10^8	130	4.90×10^9	1.40×10^{10}
(2)	1625 X 381	6.19×10^5	190	1.17×10^8	61	2.30×10^9	not to be included
		-------------		-------------		-------------	-------------
		9.09×10^5		2.28×10^8			1.40×10^{10}
						7.20×10^9→	0.72×10^{10}
							-------------
						1 =	2.12×10^{10} mm ^{4}

$$y_2 = \frac{2.28 \times 10^8}{9.09 \times 10^5} = 251$$

∴ Effective $EI_A = EI_B = 2.12 \times 10^{10}$ x EN.mm ^{2}

∴ $S_A = \frac{2bF}{EI_A} + K_{1.1} = \frac{5.86 \times 10^{10} E}{2.12 \times 10^{10} E} + 3.60 = 6.38$

and $S_B = S_A = 6.38$

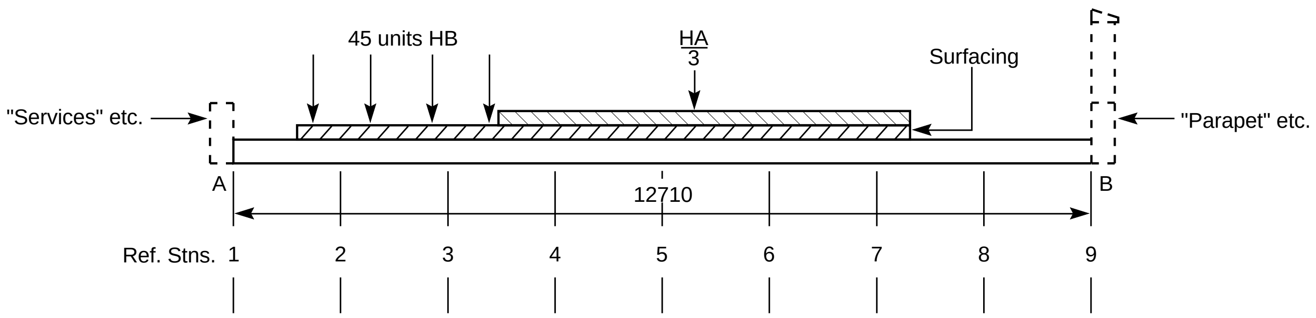
∴ $S_A \ S_B - (K_{1,2})^2 = (6.38 - 0.18)(6.38 + 0.18) = 40.5$

Allocation of Longitudinal Bending Moments to Reference Stations

The bending moments due to the various elements of loading will now be statically allocated to the 9 standard reference stations and expressed as proportions λ of the gross bending moment "M" due to 45 units of HB loading. The individual λ factors for loads due to the HB vehicle, 1/3 HA and the surfacing have been summed to give an overall λ factor for these loadings combined.

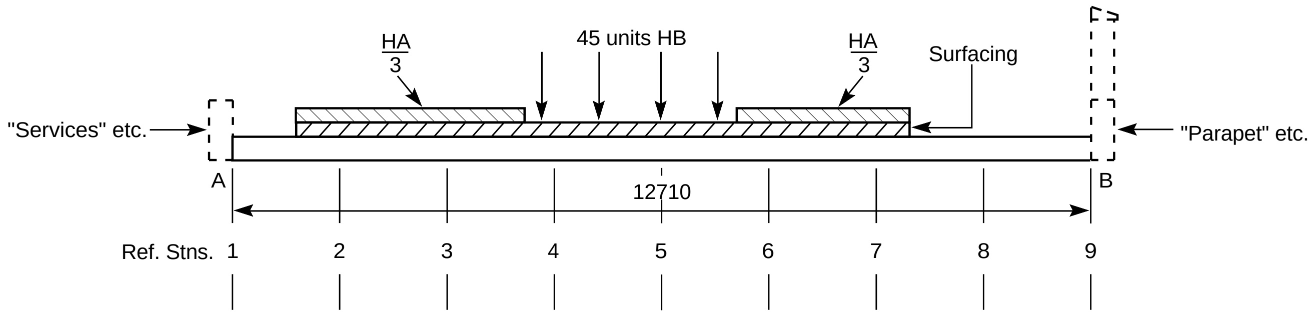
Equivalent Load Factors λ at Reference Stations for given Loading Cases

(i) *Cases IA and IB - HB Vehicle near Edge A*



Loading case	*Reference Stations*									
	1	2	3	4	5	6	7	8	9	
A. HB Vehicle	0.04	0.35	0.47	0.14						Σ = 1.0M
B. HA/3				0.03	0.08	0.08	0.03			Σ = 0.22M
C. Surfacing		0.01	0.02	0.03	0.03	0.03	0.01			Σ = 0.13M
Σ A + B + C	0.04	0.36	0.49	0.20	0.11	0.11	0.04			
D "parapet"	0.05								0.10	Σ = 0.15M

(ii) *Cases llA and llB - HB Vehicle near Centre of Deck*



Loading case	*Reference Stations*									
	1	2	3	4	5	6	7	8	9	
A. HB Vehicle			0.04	0.35	0.47	0.14				Σ = 1.0M
B. HA/3		0.03	0.08	0.04		0.04	0.03			Σ = 0.22M
C. Surfacing		0.01	0.02	0.03	0.03	0.03	0.01			Σ = 0.13M
Σ A + B + C		0.04	0.14	0.42	0.50	0.24	0.04			
D "parapet" and "services"	0.05								0.10	Σ = 0.15M

In the three following sets of tables the longitudinal bending moments in the deck beams and edge stiffening beams will be derived. In the first two sets of tables the factors λ will be applied to the moment coefficient K, obtained from the curves in Part 2 for the various elements of loading, and the resultant moment coefficients obtained for the unstiffened bridge. The ratio M_A/M and M_B/M will then be obtained from equations 1 and 2 or 5 and 6 of paragraph 10.1 (Part 1) as appropriate and used as λ's to get the resultant moment coefficient for the stiffened bridge.

In the third set of tables the λK's for the different elements of loading will be summed and converted into bending moments per metre width of deck by multiplying them by M/2b i.e. by 1810/12.71. The bending moments in the edge stiffening beams will also be obtained by summing the ratios of M_A/M and M_B/M from the previous tables and multiplying them by M(=1810kNm).

The result from these tables are plotted on the graph on page 26, which shows the maximum longitudinal bending moments across the slab for the different loading cases. Some conclusions are then drawn from the results.

Derivation of Longitudinal Bending Moments in Deck and Edge Stiffening Beams

Case I HB Vehicle Near Edge A

(i) *Unstiffened Edges*

Load case	*Load at*	λ	*Moment coefficients at reference stations* 1(A) K	λK	2 K	λK	3 K	λK	4 K	λK	5 K	AK	9(B) K	AK
	1	0.04	3.62	0.14	2.31	0.09	1.47	0.06	0.93	0.04	0.60	0.02	0.19	0.01
A	2	0.36	2.31	0.83	2.55	0.92	1.62	0.58	1.04	0.37	0.67	0.24	0.22	0.08
+	3	0.49	1.47	0.72	1.62	0.81	2.12	1.04	1.35	0.66	0.87	0.43	0.28	0.14
B	4	0.20	0.93	0.19	1.04	0.21	1.35	0.27	1.95	0.39	1.25	0.25	0.40	0.08
+	5	0.11	0.60	0.07	0.67	0.07	0.87	0.09	1.25	0.14	1.89	0.21	0.60	0.07
C	6	0.11	0.40	0.04	0.45	0.05	0.58	0.06	0.83	0.09	1.25	0.14	0.93	0.10
	7	0.04	0.28	0.01	0.30	0.01	0.40	0.02	0.58	0.02	0.87	0.03	1.47	0.06
	8	0	0.22	-	0.25	-	0.30	-	0.45	-	0.67	-	2.31	-
	9	0	0.19	-	0.22	-	0.28	-	0.40	-	0.60	-	3.62	-
	ΣλK	-	(KA)	2.00	-	2.16	-	2.12	-	1.71	-	1.32	(KB)	0.54
D	1	0.05	3.62	0.18	2.31	0.12	1.47	0.07	0.93	0.05	0.60	0.03	0.19	0.01
	9	0.10	0.19	0.02	0.22	0.02	0.28	0.03	0.40	0.04	0.60	0.06	3.62	0.36
	ΣλK			0.20	-	0.14	-	0.10	-	0.09	-	0.09	-	0.37

(ii) *Stiffened Edges*
Edge Stiffening Factors:
(10.1 (5) & (6))

$$\frac{M_A}{M} = \frac{2.0}{6.38} = 0.31; \frac{M_B}{M} = \frac{0.54}{6.38} = 0.08$$

Load case	Load at	λ	*Moment coefficients at reference stations* 1(A) K	λK	2 K	λK	3 K	λK	4 K	λK	5 K	λK	9 (B) K	λK
A+B+C(unstiffened)				2.00		2.16		2.12		1.71		1.32		0.54
M_A/M	1	-0.31	3.62	-1.13	2.31	-0.72	1.47	-0.46	0.93	-0.29	0.60	-0.19	0.19	-0.06
M_B/M	9	-0.08	0.19	-0.01	0.22	-0.02	0.28	-0.02	0.40	-0.03	0.60	-0.05	3.62	-0.29
	∑λK	-	-	0.86	-	1.42	-	1.64	-	1.39		1.08	-	0.19
D	1&9			0.20		0.14		0.10		0.09		0.09		0.37

Derivation of Longitudinal Bending Moments in Deck and Edge Stiffening Beams
Case II HB Vehicle Near Centre of Deck

(i) Unstiffened Edges

Load case	*Load at*	λ	*Moment coefficients at reference stations*											
			1(A)		3		4		5		6		9(B)	
			K	λK	K	λK	K	λK	K	λK	K	λK	K	λK
	1	0.	3.62	-	1.47	-	0.93	-	0.60	-	0.40	-	0.19	-
A	2	0.04	2.31	0.09	1.62	0.06	1.04	0.04	0.67	0.03	0.45	0.02	0.22	0.01
+	3	0.14	1.47	0.21	2.12	0.30	1.35	0.19	0.87	0.12	0.58	0.08	0.28	0.04
B	4	0.42	0.93	0.39	1.35	0.57	1.95	0.82	1.25	0.53	0.83	0.35	0.40	0.17
+	5	0.50	0.60	0.30	0.87	0.43	1.25	0.62	1.89	0.94	1.25	0.62	0.60	0.30
C	6	0.21	0.40	0.08	0.58	0.12	0.83	0.17	1.25	0.26	1.95	0.41	0.93	0.20
	7	0.04	0.28	0.01	0.40	0.02	0.58	0.02	0.87	0.04	1.35	0.05	1.47	0.06
	8	0	0.22	-	0.30	-	0.45	-	0.67	-	1.04	-	2.31	-
	9	0	0.19	-	0.28	-	0.40	-	0.60	-	0.92	-	3.62	-
	ΣλK	-	(KA)	1.08	-	1.50	-	1.86	-	1.92	-	1.53	(KB)	0.78
D	1	0.05	3.62	0.18	1.47	0.07	0.93	0.05	0.60	0.03	0.40	0.02	0.19	0.01
	9	0.10	0.19	0.02	0.28	0.03	0.40	0.04	0.60	0.06	0.93	0.09	3.62	0.36
	ΣλK	-	-	0.20	-	0.10	-	0.09	-	0.09	-	0.11	-	0.37

(ii) Stiffened Edges
Edge Stiffening Factors::
(10.1 (5) and (6))

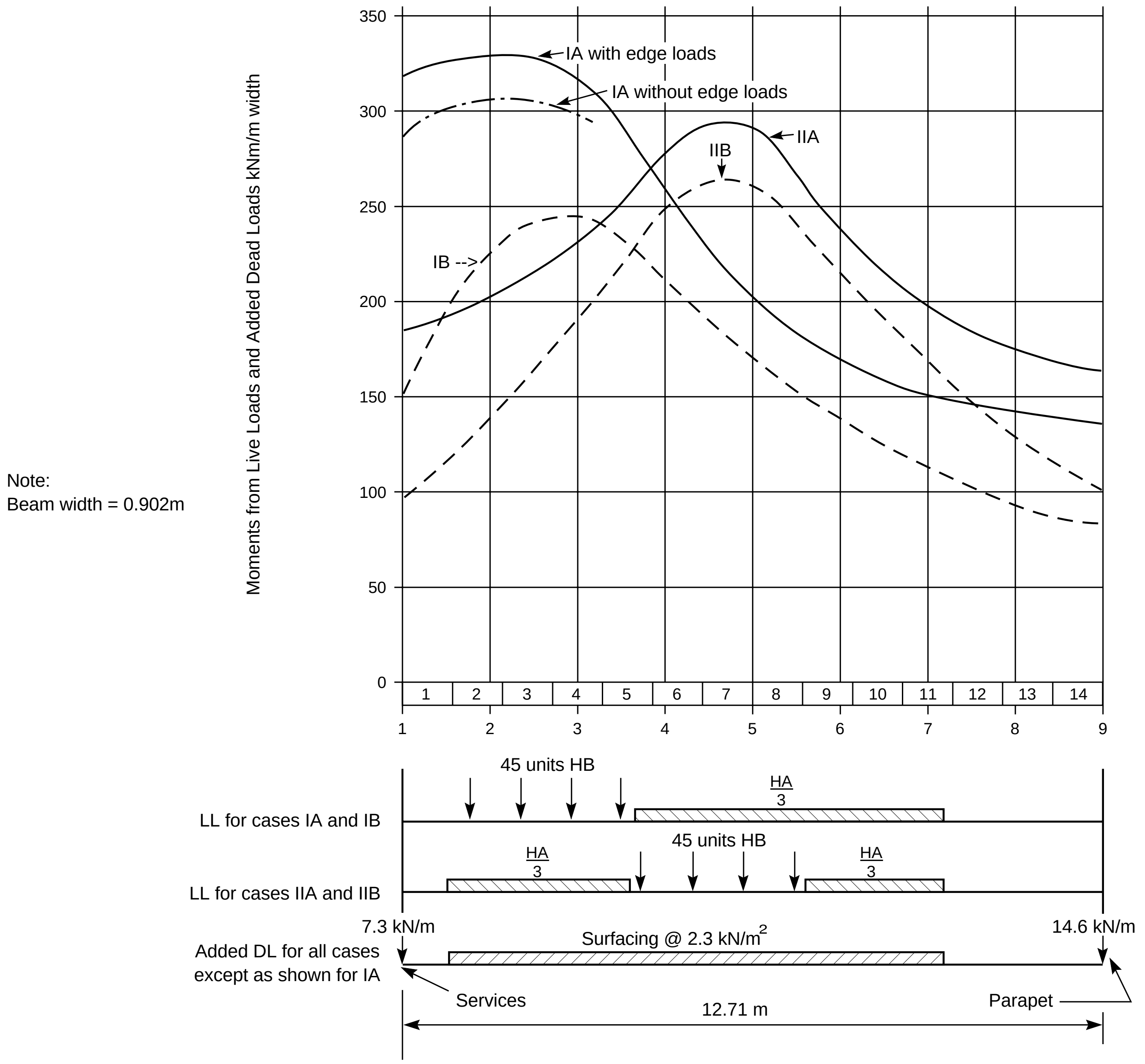
$$\frac{M_A}{M} = \frac{1.08}{6.38} = 0.17; \quad \frac{M_B}{M} = \frac{0.78}{6.38} = 0.12$$

Load case	Load at	λ	*Moment coefficients at reference stations*											
			1(A)		3		4		5		6		9 (B)	
			K	λK	K	AK	K	λK	K	λK	K	λK	K	λK
A+B+C(unstiffened)				1.08		1.50		1.86		1.92		1.53		0.78
M_A/M	1	-0.17	3.62	-0.62	1.47	-0.25	1.93	-0.16	0.60	-0.10	0.40	-0.07	0.19	-0.03
M_B/M	9	-0.12	0.19	-0.02	0.28	-0.03	0.40	-0.05	0.60	-0.07	0.93	-0.11	3.62	-0.44
	∑λK	-	-	0.44	-	1.22	-	1.65	-	1.75		1.35	-	0.34
D	1&9			0.20		0.10		0.09		0.09		0.11		0.37

Longitudinal Bending Moments in Deck and Stiffening Beams

Description of loads			*Bending moments in deck beams in kNm/m at reference stations*						
			1	2	3	4	5	6	9
Load Case I	IA unstiffened	∑λK BM	2.20 314	2.30 328	2.22 316	1.80 256	1.41 200	- -	0.91 129
Loads A + B + C + D	IB stiffened	∑λK BM	1.06 151	1.56 222	1.74 248	1.48 210	1.17 166	- -	0.56 80
Loads Case II	IIA unstiffened	∑λK BM	1.28 182	- -	1.60 226	1.95 278	2.01 286	1.64 233	1.15 164
Loads A + B + C + D	IIB stiffened	∑λK BM	0.64 91	- -	1.32 188	1.74 248	1.84 262	1.46 208	0.71 101

Description of loads	*Bending moments in edge stiffening beams*
Load Case I Loads A + B + C + D	$M_A = \sum M_A/M \times M = 0.31 \times 1810 = 562$ kNm $M_B = \sum M_B/M \times M = 0.08 \times 1810 = 145$ kNm
Load Case II Loads A + B + C + D	MA and MB not calculated as Case I is clearly the worst case. Both beams should be designed for a BM of 562 kNm. They have been assumed constructed on the unpropped slab edge, so their own weight BM has already been provided for - contrary to the case of the deck beams.



MAXIMUM MOMENTS ACROSS SLAB kNm/m WIDTH

Conclusions

1. When there is no edge stiffening, the maximum bending moment produced is greater with the vehicle eccentric (Case IA) than with it central (Case IIA). It is possible that an even greater BM could be produced with the vehicle eccentric to the other side since the "parapet, etc load is greater than the "services", etc. load. However, with the vehicle not being able to approach so close to the right-hand edge, this seems unlikely. The maximum BM under Case IA, occurring in Beam 2, is 330 x 0.902 = 298kNm.

2. With the postulated edge stiffening effective, the greatest BM per beam s reduced by about 20%, to 239kNm occurring in Beam 6 or Beam 7 (Case IIB) but judging from Case IB which produces a maximum BM of 224kNm in Beam 4, most of the beams could be subject to a similar BM. Here again, a shift of the vehicle towards the right might produce a marginally greater BM than 239kNm.

3. The maximum BM in any beam without any edge loading or edge stiffening operative is about 280kNm (occurring in Case IA, Beam 3). Since the maximum BM with the edge loads and edge stiffening effective, at 239kNm, is less than this, the edge stiffening structures more than "carry themselves". Had the stiffening effect been less potent or the weight been much greater, this might not have been the case.

Note: At this stage the amount of pre-stress required and the depth of slab should be checked and decisions made as to whether to post-tension or pre-tension, to debond and/or deflect tendons and whether to make the beams of uniform section or hog-backed or of other profile.

For beams of variable section the distribution analyses may be done using average bending and torsional stiffness - with moments, torques, etc., being derived as if the beams were uniform. The stresses should then be derived using the actual section properties. However, it has been found that for hog-backed beams with end depths not less than two-thirds the mid-span depth, the use of mid-span values of EI and GJ instead of average values has tended to increase β very slightly. This was on the side of safety in respect of longitudinal BM's and gave a negligible worsening (about 3%) in respect of transverse shears.

Beams of uniform section are assumed in this example.

Longitudinal Shear

The beams should be designed to meet the worst combination of longitudinal shear and torque effects. Since maximum longitudinal shear is produced at the ends, with three axles of the HB vehicle on the span, while maximum end torque is produced with two axles near mid-span, it will be necessary to try different positions of the vehicle, and, since torque depends on transverse shear, it will be necessary to evaluate this first.

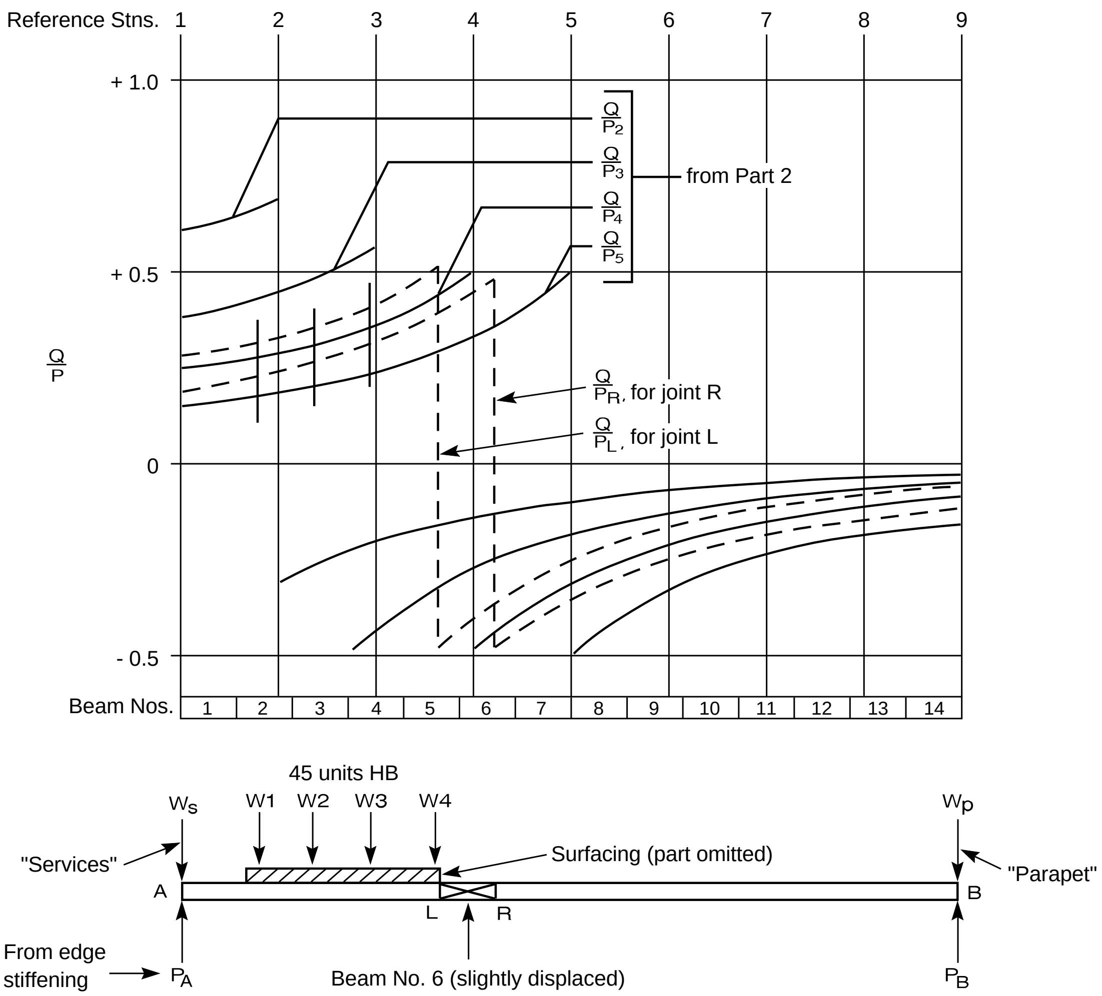
Transverse Shear

The worst longitudinal position of loading A (the HB vehicle) for transverse shear is the same as that which produces max longitudinal bending moment, i.e. with one bogey near mid-span, and the worst transverse position is as close as possible to an edge, i.e. the Case I position, but this does not necessarily apply if there is edge stiffening. For this example, the Case 1 position only will be considered, and Method (b) of 6.4 will be used. In accordance with 6.3 and 9.1 the beam which has the greatest transverse shears along its edges will be that which lies immediately adjacent to the inner edge of the vehicle. This is Beam No.6 - however, it will be assumed to be display slightly to the left from its true position so as to bring its left-side joint immediately under the line of action of wheel line W4 in the diagram since this will produce the worst possible shears. The transverse shears for joints L and R at the left and right sides of this beam (see diagram) will be derived for each loading (except as noted below), as both will be required for evaluating the maximum torque.

Loading B (HA/3) will be omitted in accordance with 6.3 (inspection of the diagram shows that the ordinates of Q/P_L under it would be entirely negative and those of Q/P_R almost entirely so).

With regard to loading C, the Q/P_L ordinates to the right of joint L are negative, while those to the left are positive. It will be assumed that the surfacing to the right of this joint has been stripped for repair.

With reference to loading D, positive ordinates apply for the "services", etc., whereas negative ordinates apply for the "parapet", etc. Both will be included however, since for the edge-stiffened case, extra load must be present at each edge, out of which the stiffening structure is formed.



INFLUENCE LINES FOR TRANSVERSE SHEAR AT REFERENCE STATIONS 2-5.

Description	*Transverse shears at joint L* $P \times (Q/P)_L = Q_L$		*Transverse shears at joint R* $P \times (Q/P)_R = Q_R$	
Loading A (HB Vehicle) $\pi^2/L^2 M_{max} = \pi^2/9.75^2 \times 1810 = 188$kN/m = P	kN/m		kN/m	
From diagram, Q/P (av.) under W_1 to W_4 for				
joint L = 1/4 (0.35 + 0.38 + 0.44 + 0.55) = 0.43				
joint R = 1/4 (0.25 + 0.29 + 0.32 + 0.39) = 0.31				
∴ Distributed transverse shear =	= 188 x 0.43 =	80.8	188 x 0.31 =	58.2
Local shear, from 6.1 = 112.5 x (0.41 - 1.75/19 x 0.25) =		43.3		43.3
∴ Total transverse shear for unstiffened deck =		124.1		101.5
Edge stiffening upward reactions at slab edges:				
$*Q_A = M_A/MO = 0.28 \times 188 = 52.8$kN/m	- 52.8 x 0.32 =	-16.9	-52.8 x 0.23	-12.2
$*Q_B = M_B/MP = 0.04 \times 188 = 7.5$kN/M	-7.5 x -0.08 =	0.8 ------	-7.5 x -0.11	0.8 ------
∴ Total transverse shear for stiffened deck =		107.8		90.1
Loading B (HA/3) not acting		0		0

*Calculation of M_A and M_B due to HB vehicle (Case A)

K_A = (0.04 x 3.62 + 0.35 x 2.31 + 0.47 x 1.47 + 0.14 x 0.93)

= (0.14 + 0.81 + 0.69 + 0.13) = 1.77

∴ M_A/M = 1.77/6.38 = 0.28

K_B = (0.04 x 0.19 + 0.35 x 0.22 + 0.47 x 0.28 + 0.14 x 0.40)

= (0.01 + 0.07 + 0.13 + 0.06) = 0.27

∴ M_B/M = 0.27/6.38 = 0.04

Description	*Transverse shears at joint L* $P \times (Q/P)_L = Q_L$	*Transverse shears at joint R* $P \times (Q/P)_R = Q_R$
Loading C (surfacing - part omitted) Width of surfacing retained = 3.23m	kN/m	kN/m
$\pi^2/L^2 M_{max} = \pi^2/9.75^2 \times \frac{2.3 \times 3.23 \times 9.75^2}{8}$ = 9.2kN/m = P		
Q/P_L (av.) under loaded area, from inspection = 0.40		
Q/P_R (av.) under loaded area, from inspection = 0.30		
∴ Transverse shears for unstiffened deck =	9.2 x 0.40 = 3.7	9.2 x 0.30 = 2.8
Trans. shears due to edge stiffening reactions at slab edges can be taken as in same proportions as under Loading A. Re Q_A	(- 16.9/124.1 x 3.7) = -0.5	(-12.2/101.5 x 2.8) = -0.3
Re Q_B	(0.6/124.1 x 3.7) = 0	(0.8/124.1 x 2.8) = 0
∴ Transverse shears for stiffened deck =	3.2	2.5
Loading D ("services" and "parapets" etc.)		
For "services" etc., $\pi^2/L^2 M_{max}$		
= $\pi^2/9.75^2 \times 87$ = 9.0kN/m = P	9.0 x 0.32 = 2.9	9.0 x 0.23 = 2.1
For "parapet" etc., $\pi^2/L^2 M_{max}$		
= $\pi^2/9.75^2 \times 173$ = 18.0kN/m = P	18.0 x -0.08 = -1.4	18.0 x -0.11 = -2.0
\ Transverse shears for unstiffened and stiffened decks =	1.5	0.1
All loadings Transverse shear for unstiffened deck: Σ =	129.3	104.4
Transverse shear for stiffened deck: Σ =	112.5	92.7

For the stiffened deck, it should be checked that the max transverse shear at the worst slab-edge (i.e. ΣQ_A) does not exceed the transverse shear at joint L (i.e ΣQ_L = 129.3kN/M). For this purpose, full HA/3 and full surfacing loads should be included.

Loadings	$P = \pi^2/L^2 M_{max}$ kN/m	$(Q/P)_A = M_A/M$	Transverse shear at edge A $P \times (Q/P)_A = Q_A$ kN/m
A	$\pi^2/9.75^2 \times 1810 = 188$	0.28	188 x 0.28 = 53.0
B	$\pi^2/9.75^2 \times 400 = 41$	0.019*	41 x 0.019 = 0.8
C	$\pi^2/9.75^2 \times 240 = 25$	0.018*	25 x 0.018 = 0.5
All loadings			Σ = 54.3

Hence the max transverse shear at edge A is less than that at the worst internal joint (i.e. Joint L).

Transverse Shear Effects

The transverse shear steel will be assumed to be the minimum specified in 6.6 which amounts to A_S = 754 mm^2/m, placed at depth d = 305 and carried up to the joints to 30 mm below top of beam. This (allowing for a 90º bend) gives an anchorage length $l_b$of approx. 420 mm. Lever arm a = (say) 0.9d = 275mm, b = 1m and perimeter 0 = 251 mm/m.

		Unstiffened	Stiffened deck
Max shear V	kN/m	129.3	112.5
Shear stress on joint concrete V/ba	N/mm^2	0.47	0.41
(Permissible = 0.87N/mm2) (HB = 1.09)			
Local bond stress V/ao	N/mm^2	1.87	1.63
(Permissible = 1.47N/mm^2) (HB = 1.84)			
Average bond stress Vl_{bo}	N/mm^2	1.23	1.07
(Permissible = 1.00 N/mm^2) (HB = 1.25)			
Direct tensile stress on reinft. V/A_S	N/mm^2	171	149
(Permissible HB 138 x 1.25 = 172N/mm^2)			

In view of the high-stresses shown above, high-bond steel should be used for the unstiffened deck.

The low value of shear stress on the joint concrete is of interest and raises a question as to how important transverse reinforcement really is, provided there is good bond of the concrete to the beam sides.

*Calculation of M_A/M for load cases B and C.

B: K_A = (0.03 x 0.93 + 0.08 x 0.60 + 0.08 x 0.40 + 0.04 x 0.28)
= (0.028 +0.048 + 0.032 + 0.008) = 0.116

$\therefore$ M_A/M = 0.116/6.38 = 0.019.

C: K_A = (0.023 + 0.029 + 0.028 + 0.018 + 0.012 + 0.003) = 0.113

$\therefore$ M_A/M = 0.113/6.38 = 0.018

Longitudinal Shear and Torque

The worst case for principal tensile stress due to combined longitudinal shear and torque probably occurs with the vehicle at the position which produces maximum longitudinal bending. This case only will be investigated, but in a complete analysis it would be necessary to consider other cases also. Beam 6 (slightly displaced) will be the worst affected beam, and it will be assumed that no part of the longitudinal shear due to parapets and pavements will have to be resisted by this beam, but that it will be subjected to the full shear from wheels at the W4 position. This, at the beam-end, amounts to 225 x 5.325/9.75 = 123kN, and the shear due to self-weight and in-filling to 40kN. The road surfacing has been assumed removed from this beam, so the total shear = 163kN and the max. shear stress produced at N.A. level = 163 x 1.5 x 10^3/(902 x 381) = 0.71N/mm^2.

The simultaneously-acting torque shear stress at the end of Beam 6 is calculated in the following table:

	Unstiffened deck		*Stiffened deck*	
Distributed transverse shear at joint L	kN/m 129.3 - 43.3 =	86.0	112.5 - 43.3 =	69.2
Distributed transverse shear at joint R	kN/m 104.4 -43.3 =	61.1 ------- 147.1	92.7 - 43.3 =	49.4 ------- 118.6
∴ Average distrib. trans. shear over width of beam =	kN/m	73.6		59.3
∴ Average torque over width of beam = T_o = (per unit width)	kN 7.6 x span/π =	228	59.3 x span/π =	183
∴ Total end torque on Beam 6 = T_oB =	kN/m 228 x 0.908 =	206	183 x 0.908 =	166
∴ Torsional shear stress at X = $\tau_x = T_oB/Z_{TX}$ =	N/mm² 206 x 10^6 /4.33 x 10^7) =	4.75	166 x 10^6/(4.33 x 10^7) =	3.84
∴ Torsional shear stress at Y = $\tau_y = T_oB/Z_{TY}$ =	N/mm² 206 x 10^6 /3.33 x 10^7) =	6.20	166 x 10^6(3.33 x 107) =	5.00

On one side of Beam 6 the torsional shear stress at X is additional to the longitudinal shear stress.

Therefore, total shear stress at X for unstiffened deck = 4.75 + 0.71 = 5.46 N/mm²
and total shear stress at X for stiffened deck = 3.84 + 0.71 = 4.55 N/mm²

At Y, the total shear stress = the torsional shear stress.

The beam is assumed to have an effective prestress σ near its bearings of zero at the top flange, increasing to 13.78N/mm2 at the bottom flange - typical values for a pre-tensioned beam. The principal tensile stresses given by $\sqrt{\sigma^2/4 + \tau^2} - \sigma/2$ (where τ is the shear stress) are thus as follows::

	Unstiffened deck	Stiffened deck
	N/mm²	N/mm²
At top flange	6.20	5.00
At neutral axis	3.00	2.25
At bottom flange	2.36	1.61

These stresses (except perhaps at the bottom flange) are far in excess of anything permissible and since they indicate that the section would be badly cracked by the combination of longitudinal shear and torque - even if not cracked by the maximum longitudinal shear acting alone - reinforcement would be required to carry the whole of the longitudinal shear and torque, and even so it is for consideration whether principal tensile stresses in excess of twice those permissible without reinforcement could be acceptable. Assuming they are, and adopting a permissible steel stress of 138kN x 1.25 for abnormal loading, the following reinforcement is required:

To resist longitudinal shear:

$A_S(1) = 163 \times 10^6/(138 \times 1.25 \times 300) = 3150$ mm²/m (same for unstiffened and stiffened decks) i.e. 1575mm²/m per face, if 2-leg stirrups are used.

To resist torque, using the formula in 9.3 and assuming σ has the neutral axis value,

	Unstiffened deck	*Stiffened deck*
$A_S(2)$ for each beam-side	$4.75^2 \times 902 \times 10^3/6 \times 6.89 \times 138 \times 1.25$ $= 2850mm^2/m$	$3.84^2 \times 902 \times 10^3/6 \times 6.89 \times 138 \times 1.25$ $= 1860mm^2/m$
OR $A_S(3)$ for beam top and bottom	$6.20^2 \times 381 \times 10^3/6 \times 6.89 \times 138 \times 1.25$ $= 2050mm^2/m$	$5.00^2 \times 381 \times 10^3/6 \times 6.89 \times 138 \times 1.25$ $= 1340mm^2/m$

whichever is the greater.

Combining $A_S(1)$ and $A_S(2)$ we require, for each side face of the beam $4425mm^2/m$ for the unstiffened deck and $3435mm^2/m$ for the stiffened deck. This would necessitate 25mm closed stirrups at 100mm centres for the unstiffened deck and almost as much for the stiffened deck. Such stirrups, in a beam of the size postulated, and bearing in mind the transverse shear steel which would have to be placed along with it, would appear to be quite impracticable. Since the beam is subject to severe torque, there should, in addition to the closed stirrups, be substantial longitudinal bars in the top two corners of the section. At the bottom two corners, the pre-stress could probably be relied on to neutralise the longitudinal component of the principal tensile stress.

2. ALLOWING FOR JOINT STIFFNESS

Calculation of Reduction Factor

Data:

D	= 381mm	
B	= 902mm $\therefore$ D/B = 0.422 and ϕ = 0.885	
A_{st}	(bottom transverse steel) = 12mm dia at 150mm crs	= $754mm^2/n$
O	(perimeter of bottom transverse steel)	= 251mm/m
d	(effective depth to bottom transverse steel)	= 305mm
E_j	of joint concrete	= $2.75 \times 10^4 N/mm^2$
E_b	of beam concrete	= $4.14 \times 10^4 N/mm^2$
E_s	of reinforcement	= $2.06 \times \ 14^5 N/mm^2$
b_j	effective width of joint (taken as)	= 125mm (75 + 50)
m	(modular ratio for joint concrete)	= 7.5
0	(% of transverse reinforcement)	= 0.247
n_j	$\sqrt{(0.01mp)^2 + 0.02mp} - 0.01mp = 0.174$	
	$a = 1 = n_j/3 = 0.942$	

A (concrete units) = $E_j d^2/(3b_j) \times 10^3)\ (n_j{}^3 d + 3E_s/(E_j \times 10^3) > A_{st}\ (1-n_j)^2)$

$$= \frac{2.75 \times 10^4 \times 305^2}{3 \times 125 \times 10^3}\left[0.174^3 \times 305 + \frac{3.x\ 2.06 \times 10^5 \times 754 \times 826^2}{2.75 \times 10^4 \times 10^3}\right]$$

= 90000kN

G (concrete units) $= \dfrac{E_b D^3}{12\ \phi^2} = \dfrac{4.14 \times 10^4 \times 381^3}{120.885^2 \times 10^6}$

$= 2.44 \times 10^5$kNm

$$\frac{A}{G} = \frac{90000}{2.44 \times 10^5} = 0.369 \text{ per metre}$$

$$\text{Let } K = \frac{2L^2}{n^2\pi^2 B} \times \frac{A}{G} = \frac{2 \times 9.75^2 \times 0.369}{1 \times \pi^2 \times 0.902}$$

= 7.88

$\therefore$ Reduction factor = 1/(1 + K) = 1/8.88

Transverse Shear Effects

Total max transverse shear V (after reduction) for unstiffened deck (occurring as before at Joint L of Beam 6).

= 43.3 (local) + (129.3 - 43.3)/8.88 (distributed) = 53.0kN/m

Therefore all the stresses shown for the "pure hinge" calculations on page 30 should be multiplied by the factor 53.0/129.3 = 0.41. This will bring them all well within permissibles, using mild steel.

Longitudinal Shear and Torque

The longitudinal shear is unaffected by the reduction factor

With regard to torque effects, all the items tabulated on page 31 have to be multiplied by the reduction factor of 1/8.88.

∴ total shear stress at X for unstiffened deck = 4.75/8.88 + 0.71 = 1.24N/mm^2
total shear stress at X for stiffened deck = 3.84/8.88 + 0.1 = 1.14N/mm^2
total shear stress at Y for unstiffened deck = 6.20/8.88 = 0.70N/mm^2
total shear stress at Y for stiffened deck = 5.00/8.88 = 0.56N/mm^2

Assuming the same pre-stress as before, the principal tensile stresses become as follows:

	Unstiffened deck	*Stiffened deck*
	N/mm^2	N/mm^2
At top flange	0.70	0.56
At neutral axis	0.22	0.19
At bottom flange	0.03	0.01

These principal tensile stresses are now less than any of the limiting values given in Clause 9.3, Table 1, and so no torque or longitudinal shear stirrups are required. However, stirrups to comply (at least) with Section 9.4 should be provided.

Transverse Bending Moment in the Joint

From 7.2, transverse BM = M_n = 1 = 2L/π x A/G x T_o x 1/(1 + K)

= 2 x 9.75/π x 0.369 x 1/8.88 x T_o = 0.257 T_o

For all loadings, as before T_o is a maximum in Beam 6 (slightly displaced) having values as tubulated on page 31.

		Unstiffened deck		*Stiffened*	
∴ max transverse BM	=kN/m/m	0.257 x 228	= 58.7	0.257 x 183	= 47.0
∴ stress in reinft.					
= M/A_{ST}ad) = M/(754 x 0.942 x 305)	= N/mm^2	58.7 x 10^6/217000	=270	47.0 x 10^6/217000	= 216
(max permissible = 228N/mm^2, when using high-yield bars)					
The edge-stiffened deck only is viable without re-design					
∴ Stress in concrete	=N/mm^2			216 x 0.174 (7.5 x 0.826)	
(max permissible for Class 30 concrete = 10.0N/mm^2) (HB = 12.5)					= 6.06

Beam Action in the Joint

Stiffened deck only is considered

Load per bar = 216 x 113 = 24400N (at 150mm centres)
M_{max} = WL/16 = 24400 x 150/16 x 10^6 = 0.229kN/m
Q_{max} = W/2 = 24400/2 x 10^3 = 12.2kN
For a 25mm dia longitudinal bar A_S = 491 mm^2
Section modulus = $\pi R^3/4 = \pi \times 12.5^3/4$ = 1530mm^3
Bending stress = 0.229 x 10^6/1530 = 150N/mm^2
(Permissible as BS 153 Pt.3B, Table 3, pins, 208.5 x 1.25 = 260N/mm^2)
Shear stress = 13.2 x 10^3/491 = 26.9N/mm^2
(Permissible as above for pins, 100.4 x 1.25 = 125N/mm^2.)

FURTHER DETAILS ON THE CALCULATION OF LONGITUDINAL MOMENTS AND TRANSVERSE SHEARS

1. Longitudinal Moments

The effect of considering more terms than the first in the series, or indeed the whole sum, is to increase the influence surface ordinates near the point at which the bending moment is considered, while decreasing them at all other points. If, therefore, groups of loads are involved, which are wide spaced in relation to the peak of the influence surface, the total effect, when calculated from the influence surface, will be very nearly the same as that due to the first harmonic. The actual influence surface for bending moments can only be obtained easily for an infinite slab. A comparison with this shows that the difference when dealing with two bogies is less than 5%.

In practice this difference will be reduced still further, since the peak value of the influence surface for an actual wheel with finite contact area and dispersion of load through the slab will be smaller than for the corresponding hypothetical point load.

The difference between the actual peak value for a wheel load on a finite slab and the first harmonic for the corresponding hypothetical point load will therefore be even less than the difference between the peak value for the point load and the first harmonic and may be neglected except where (as in the case of small areas of roadway) only one or two wheel loads have to be considered.

2. Transverse Shear

The transverse shears which occur in this type of slab are due to the overall distribution of loads plus local effects in the immediate neighbourhood of a wheel. The theory for the analysis of these slabs, like that used for ordinary slabs, does not take into account the thickness and therefore does not allow for the distribution of the load in a vertical direction. Any calculation involving a vertical point load therefore leads to infinitely large stresses immediately under that load. These are avoided by the realistic assumption that "point" loads are in fact applied through an area which is partly given by the actual area of contact of the wheel, and partly by the dispersion of vertical stresses through the depth of the slab.

The shear stresses, due to wheel loads in their immediate vicinity, have been calculated on the assumption that this dispersion is equivalent to distribution of the load over a distance equal to twice the depth of the slab. In short spans, the width of the wheel is significant in relation to the slope of the influence surface, which causes a further reduction in local stresses. As spans increase this becomes smaller and can be neglected at 8m, but the local shear is reduced because the depth of slab increases. Since the depth is assumed to be 1/25 of the span, this accounts for the linear interpolation between 8m, and 27m in paragraph 6.1. The values of the local shear were obtained from influence surfaces for this shear on an infinitely wide slab. It can be shown that when the difference between this local shear on an infinite slab, and the value obtained by considering its first harmonic shears is added to the first harmonic shears on a finite slab, the result is not greatly affected when this exercise is repeated using the higher harmonics.

COMPARISON OF COMPUTER AND HAND METHODS OF ANALYSIS

Mid-Span Longitudinal Moments in Shear Key Deck
Example from Part 3

Transverse station position	Moment due to surfacing		Moment due to ⅓ HA loading		Moment due to HB loading	
	Hand method	*Dundee* program*	*Hand method*	*Dundee program*	*Hand method*	*Dundee program*
-b	18.0	19.0	17.0	17.7	246..0	255.0
-3b/4	21.0	20.6	21.0	19.6	265.0	271.0
-b/2	22.0	22.0	27.0	26.2	250.0	252.0
-b/4	22.0	22.3	37.0	39.0	179.0	182.0
0	21.0	21.6	43.0	44.7	116.0	112.0
b/4	19.0	19.6	42.0	43.8	72.0	72.0
b/2	16.0	16.0	35.0	35.7	53.0	48.0
3b/4	12.0	12.4	27.0	26.6	46.0	37.0
b	10.0	11.2	20.0	22.6	38.0	33.0

* Computer Program for Bridge Decks with Discrete Column Supports developed at the University of Dundee.